Math 308 - Summer 2018

Selected Solutions to HW set 1

Problems 10, 22, 26, 36, 45*, and 48 were graded for HW 1.

Disclaimer: If you have questions about any of the other problems, see me in office hours. Consider all problems important, not just the ones I provide solutions for. Also consider it important to do *more* than what is required for homework.

10. Give examples of three sets A, B and C such that

There are many possible answers for this problem, but some are:

(a) $A \subseteq B \subset C$

Choose $A = B = \{1\}$, and $C = \{1,2\}$.

(b) $A \in B$, $B \in C$ and $A \notin C$

Choose $A = \{1\}$, $B = \{\{1\}, 2\}$, and $C = \{B, 3\} = \{\{\{1\}, 2\}, 3\}$.

(c) $A \in B$ and $A \subset C$.

Choose $A = \emptyset$, $B = C = \{\emptyset\}$.

22. Let $U = \{1,3, \dots, 15\}$ be the universal set, $A = \{1,5,9,13\}$, and $B = \{3,9,15\}$. Determine the following:

(a) $A \cup B$

$A \cup B = \{1,3,5,9,13,15\}$

(b) $A \cap B$

$A \cap B = \{9\}$

(c) $A - B$

$A - B = \{1,5,13\}$

(d) $B - A$

$B - A = \{3,15\}$

(e) $\bar{A}$

$\bar{A} = U - A = \{3,7,11,15\}$

(f) $A \cap \bar{B}$

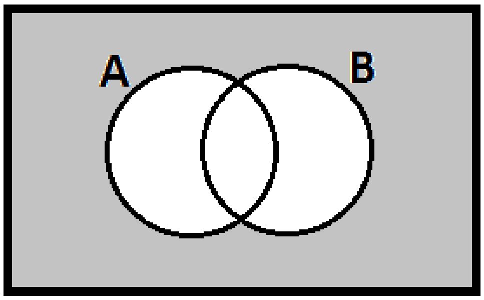
$A \cap \bar{B} = \{1,5,13\}$

26. Let U be a universal set and let A and B be two subsets of of U. Draw a Venn diagram for each of the following sets.

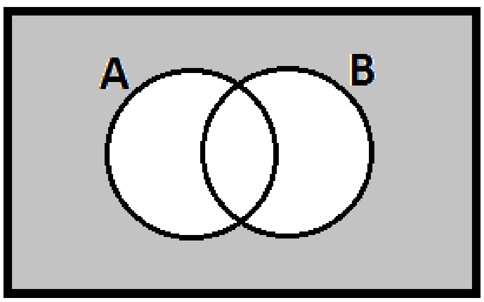
(a) $\overline{A \cup B}$ (b) $\bar{A} \cap \bar{B}$ (c) $\overline{A \cap B}$ (d) $\bar{A} \cup \bar{B}$

The part of each diagram that represents where members of the set are located is shaded grey.

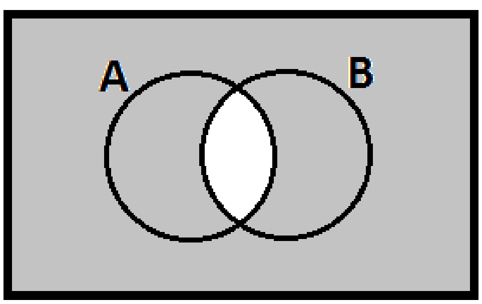
(a) $\overline{A \cup B}$



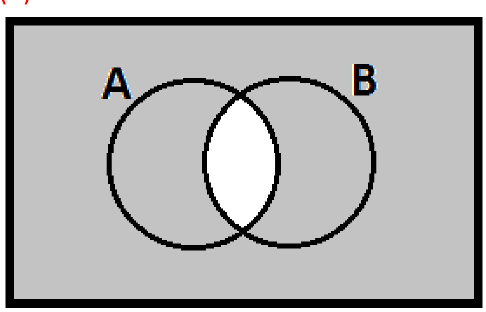
(b) $\bar{A} \cap \bar{B}$



(c) $\overline{A \cap B}$



(d) $\bar{A} \cup \bar{B}$



Note: To graph an intersection of sets, shade each section related to each set in a different pattern. Where ALL patterns overlap is the intersection. Taking all sections together that have ANY shading at all gives the intersection. To find a compliment, first shade the set, then the regions that have NO shading are in the complement. You may then decide to invert the picture and shade the non-shaded parts and erase the shading of the shaded parts.

What can be said about parts (a) and (b)? parts (c) and (d)?

(a) and (b) are the same. So are (c) and (d). The diagrams suggest (not prove) that $\overline{A \cup B} = \bar{A} \cap \bar{B}$ and $\overline{A \cap B} = \bar{A} \cup \bar{B}$. If you think these rules look familiar, you're not wrong. These are the DeMorgan's laws for sets. Look back at our DeMorgan's laws for statements and note that the complements behave like negations if the sets represented statements.

36. For a real number r, define S_r to be the interval $[r-1, r+2]$. Let $A = \{1,3,4\}$. Determine $\bigcup_{\alpha \in A} S_\alpha$ and $\bigcap_{\alpha \in A} S_\alpha$.

$$\bigcup_{\alpha \in A} S_\alpha = S_1 \cup S_3 \cup S_4 = [0,3] \cup [2,5] \cup [3,6] = [0,6].$$

$$\bigcap_{\alpha \in A} S_\alpha = S_1 \cap S_3 \cap S_4 = [0,3] \cap [2,5] \cap [3,6] = \{3\}.$$

It may help to sketch the intervals associated with each S_α.

45. For $n \in \mathbb{N}$, let $A_n = \left(-\frac{1}{n}, 2 - \frac{1}{n}\right)$. Determine $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$.

Again, it would help to sketch the intervals here:

$$\bigcup_{n \in \mathbb{N}} A_n = A_1 \cup A_2 \cup A_3 \cup \cdots = (-1,1) \cup \left(-\frac{1}{2}, \frac{3}{2}\right) \cup \left(-\frac{1}{3}, \frac{5}{3}\right) \cup \cdots = (-1,2).$$

$$\bigcap_{n \in \mathbb{N}} A_n = A_1 \cap A_2 \cap A_3 \cap \cdots = (-1,1) \cap \left(-\frac{1}{2}, \frac{3}{2}\right) \cap \left(-\frac{1}{3}, \frac{5}{3}\right) \cap \cdots = [0,1].$$

48. Let $A = \{1,2,3,4,5,6\}$. Give an example of a partition S of A such that $|S| = 3$.

Many examples, here's one: $S = \{\{1\}, \{2\}, \{3,4,5,6\}\}$. Note that the sets in S are disjoint and their union is A.

45. was not a homework problem. I did the solution by accident but decided to leave it in case anyone attempted it.