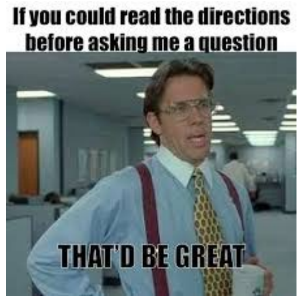
Math 212 RS2 Test 1

Quarantine, day 58.

Name: ____

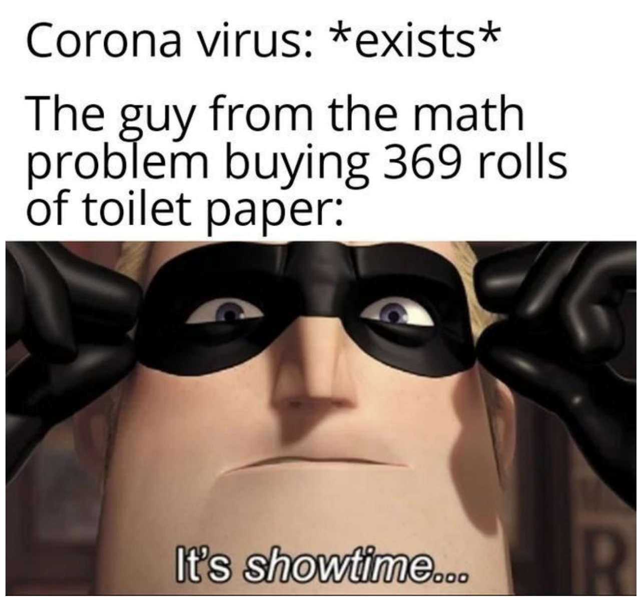
Note that both sides of each page may have printed material.



Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic!**
3. Complete all problems in the actual test. Bonus problems are, of course, optional, and will only be counted if all other problems are attempted.
4. **You have 90 minutes to complete the test.**
5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
6. Write neatly so that I am able to follow your sequence of steps and box, or otherwise indicate, your answers. Solutions with no indicated answer or several contradictory answers will be considered incorrect.
7. Read through the exam and complete the problems that are easy (for you) first!
8. You are NOT allowed to use notes, calculators, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
9. In fact, **cell phones should be out of sight! If you are caught with a cellphone you will be asked to leave the exam and you'll be given a zero. That goes for smart watches too!**
10. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

Other than that, have fun and good luck!



1. Compute the following integrals (5 points each):

$(a)\ \displaystyle\int \tan^3\theta\,\sec^5\theta\ d\theta$

$(b)\ \displaystyle\int_0^{1/2} \arcsin x\ dx$

$$(c)\ \int \frac{x^2 + x - 3}{x^2 + x - 2}\, dx$$

2. Approximate the integral below using 4 subintervals and:

$$\int_{-1}^{1} (x^3 + 1)\, dx$$

(a) The Simpson's rule (5 points):

(b) Compare your estimate with the exact value of the integral. (5 points)

3. For each of the integrals below determine, with justification, whether they converge or diverge. If convergent, say what they converge to (10 points each)

$$(a)\ \int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx$$

$$(b)\ \int_0^1 \frac{\sec^2 x}{x^3}\,dx$$

4. (5 points each) For each of the following series, state, with justification, whether the series converges absolutely, converges conditionally, or diverges.

$$(a)\ \sum_{n=1}^{\infty}(-1)^n \frac{n+4}{\sqrt{n^4+2n^2+2}}$$

$$(b)\ \sum_{n=1}^{\infty}\frac{\sqrt{n}\,e^n}{n!}$$

$$(c)\ \sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$$

5. (10 points) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-5)^n}{2^n \sqrt[3]{n}}$

6. (a) (5 points) Use a known power series to find a power series for

$$f(x) = x^2 e^{-x^2}$$

(b) (10 points) Using your answer above, approximate $\int_0^{1/10} x^2 e^{-x^2}\, dx$ using a sum of two terms.

(c) (5 points) Give an upper bound for the error in your approximation above, and justify your choice.

7. (10 points) Suppose $f(x) = \sqrt[3]{x}$. Find the third degree Taylor polynomial of $f(x)$ centered at $x = 1$.

Bonus: (Bonus points will only be counted if all other problems are attempted).

1. (a) (2 points) Convert $(x, y) = (-2, -2)$ to polar coordinates (r, θ)

   (b) (2 points) Convert $(r, \theta) = \left(2, \frac{-5\pi}{6}\right)$ to rectangular coordinates (x, y)

   (c) (2 points) Describe the following region (including its boundary) using polar coordinate inequalities. Include a sketch. *The region in the first quadrant, below the line $y = x$, outside the circle of radius 1 but inside the circle of radius 5.*

2. (2 points each part) Let $\vec{a} = < 2, -1, -1 >$ and $\vec{b} = < 3,0,2 >$. Compute:

   $(a)\ \vec{a} \cdot \vec{b}$ $\qquad$ $(b)\ \ \vec{a} \times \vec{b}$

3. (1 point each) Let $\vec{a}$ and $\vec{b}$ be 3-dimensional vectors. Define the following phrases using dot products or cross products (whichever is appropriate).

   (a) $\vec{a}$ and $\vec{b}$ are parallel: ____

   (b) $\vec{a}$ and $\vec{b}$ are orthogonal: ____

4. (8 points) Identify the conic section and its center and sketch it: $x^2 + 4y + 7 = 6x - 2y^2$