

8 + 2 Bonus points possible!

Name: ANSWERS

Instructions: No calculators. Use your own scrap paper and write your answers in the space provided.

1. Suppose $\int_a^b f(x) dx$ is an integral in which $f(x)$ is a rational expression of trig functions.

(a) State the traditional Weierstrass substitution: $u = \underline{\tan \frac{x}{2}}$

(b) Using this substitution, derive or state $\sin x = \underline{\frac{2u}{1+u^2}}$ as a function of u

(c) Using this substitution, derive or state $\cos x = \underline{\frac{1-u^2}{1+u^2}}$ as a function of u

(d) Using this substitution, derive or state $dx = \underline{\frac{2}{1+u^2} du}$ in terms of du

(e) Compute: $\int \frac{1}{1 + \cos x + \sin x} dx = \underline{\ln \left| \tan \frac{x}{2} + 1 \right| + C}$

2. Integrate the following:

(a) $\int \frac{x}{(1-3x)^3} dx = \underline{\frac{1}{18(1-3x)^2} - \frac{1}{9(1-3x)} + C}$ (b) $\int \frac{x^2 + 5x - 2}{x^2 - x + 1} dx = \underline{x + 3 \ln |x^2 - x + 1| + C}$

(c) $\int \frac{2}{x^2 + x - 2} dx = \underline{\frac{2}{3} \ln |x-1| - \frac{2}{3} \ln |x+2| + C}$ OR $\underline{\frac{2}{3} \ln \left| \frac{x-1}{x+2} \right| + C}$

Bonus:

1. In approximating the integral $\int_a^b f(x) dx$ with n subintervals, define what Δx is.

$$\Delta x = \underline{\frac{b-a}{n}}$$

2. Name three numerical integration rules used to approximate definite integrals:

(Any three of) Left hand rule, Right hand rule,

Midpoint rule, Trapezoid rule, Simpson's rule.