Math 212 RS2 Quiz 4A

February 18, 2020

8 + 2 Bonus points possible!

Name: ANSWERS

Instructions: No calculators. Use your own scrap paper and write your answers in the space provided.

1. Suppose $\int_a^b f(x)\ dx$ is an integral in which $f(x)$ is a rational expression of trig functions.

   (a) State the traditional Weierstrass substitution: $u =$ $\tan\frac{x}{2}$

   (b) Using this substitution, derive or state $dx =$ $\frac{2}{1+u^2}\,du$ in terms of du

   (c) Using this substitution, derive or state $\sin x =$ $\frac{2u}{1+u^2}$ as a function of u

   (d) Using this substitution, derive or state $\cos x =$ $\frac{1-u^2}{1+u^2}$ as a function of u

   (e) Compute: $\int \frac{1}{\sin x + \cos x + 1} dx =$ $\ln\left|\tan\frac{x}{2}+1\right| + C$

2. Integrate the following:

   (a) $\int \frac{x}{(1-2x)^3}\ dx =$ $\frac{1}{8(1-2x)^2} - \frac{1}{4(1-2x)} + C$

   (b) $\int \frac{x^2+5x+3}{x^2+x+1}\ dx =$ $x + 2\ln|x^2+x+1| + C$

   (c) $\int \frac{2}{x^2-x-2}\ dx =$ $\frac{2}{3}\ln|x-2| - \frac{2}{3}\ln|x+1| + C$ OR $\frac{2}{3}\ln\left|\frac{x-2}{x+1}\right| + C$

Bonus:

1. In approximating the integral $\int_a^b f(x)\ dx$ with n subintervals, define what Δx is.

   $\Delta x =$ $\frac{b-a}{n}$

2. Name three numerical integration rules used to approximate definite integrals:

   (Any three of) Lefthand rule, Righthand rule, Midpoint rule, Trapezoid rule, Simpson's rule.