Math 212 RS2 Quiz 3B
February 11, 2020

(16) + 3 Bonus points possible

Name: ANSWERS

Instructions: No calculators. Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a) $\int \sec^3 x\, dx =$ $\frac{1}{2}(\sec x\tan x + \ln|\sec x+\tan x|)+C$ (b) $\int \sec x\, dx =$ $\ln|\sec x+\tan x|+C$

(c) $\int \frac{1}{\sqrt{1-x^2}}dx =$ $\sin^{-1}x+C$ (d) $\int \ln x\, dx =$ $x\ln x - x + C$

(e) $\int \frac{1}{1+x^2}dx =$ $\tan^{-1}x+C$ (f) $\int \tan x\, dx =$ $\ln|\sec x|+C$ OR $-\ln|\cos x|+C$

2. Complete the following table of trig substitutions (the first row is an example):

Expression	Substitution	Identity
a^2+x^2	$x = a\tan\theta$	$1+\tan^2\theta = \sec^2\theta$
x^2-a^2	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$
a^2-x^2	$x = a\sin\theta$	$1-\sin^2\theta = \cos^2\theta$

3. Integrate the following:

(a) $\int \sin^3 x\cos^4 x\, dx =$ $\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5}+C$ (b) $\int \cos^2 x\, dx =$ $\frac{1}{2}\left(x+\frac{1}{2}\sin 2x\right)+C$

(c) $\int \tan^3\theta\sec^4\theta\, d\theta =$ $\frac{\tan^4\theta}{4}+\frac{\tan^6\theta}{6}+C$ OR $\frac{\sec^6\theta}{6}-\frac{\sec^4\theta}{4}+C$ (d) $\int \frac{x^2}{\sqrt{1-x^2}}dx =$ $\frac{1}{2}\left(\sin^{-1}x - x\sqrt{1-x^2}\right)+C$

(e) $\int x^2\sin x\, dx =$ $-x^2\cos x + 2x\sin x + 2\cos x + C$ (f) $\int \frac{\sin 2x}{1+\cos^4 x}dx =$ $-\tan^{-1}(\cos^2 x)+C$

Bonus:

1. $\int \frac{x^2-1}{x^2+1}dx =$ $x - 2\tan^{-1}x + C$ (b) $\int \frac{1}{x^2+x-6}dx =$ $\frac{1}{5}\left(\ln|x-2|-\ln|x+3|\right)+C$

2. Write down the partial fractions decomposition of $\frac{12}{x^2(x^2+9)^2(x^2-1)}$. You may use $A, B, C, \ldots$ for the arbitrary constants. You need not find the values of the arbitrary constants.

$\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+9}+\frac{Ex+F}{(x^2+9)^2}+\frac{G}{x-1}+\frac{H}{x+1}$