Math 212 RS2 Quiz 3A
February 11, 2020

16 + 3 Bonus points possible

Name: ANSWERS

Instructions: No calculators. Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a) $\int \frac{1}{\sqrt{1-x^2}}\,dx =$ $\sin^{-1}x + C$ (b) $\int \frac{1}{1+x^2}\,dx =$ $\tan^{-1}x + C$

(c) $\int \sec^3 x\,dx =$ $\frac{1}{2}(\sec x\tan x + \ln|\sec x + \tan x|) + C$ (d) $\int \ln x\,dx =$ $x\ln x - x + C$

(e) $\int \tan x\,dx =$ $\ln|\sec x| + C$ or $-\ln|\cos x| + C$ (f) $\int \sec x\,dx =$ $\ln|\sec x + \tan x| + C$

2. Complete the following table of trig substitutions (the first row is an example):

Expression	Substitution	Identity
$a^2 - x^2$	$x = a\sin\theta$ *or* $x = a\cos\theta$	$1 - \sin^2\theta = \cos^2\theta$ *or* $1 - \cos^2\theta = \sin^2\theta$
$x^2 - a^2$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$
$x^2 + a^2$	$x = a\tan\theta$	$\tan^2\theta + 1 = \sec^2\theta$

3. Integrate the following:

(a) $\int \sin^3 x\cos^4 x\,dx =$ $\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$ (b) $\int \sin^2 x\,dx =$ $\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

(c) $\int \tan^5\theta\sec^3\theta\,d\theta =$ $\frac{\sec^7\theta}{7} - \frac{2\sec^5\theta}{5} + \frac{\sec^3\theta}{3} + C$ (d) $\int \frac{x^2}{\sqrt{1+x^2}}\,dx =$ $\frac{1}{2}\left(x\sqrt{1+x^2} - \ln\left|\sqrt{1+x^2} + x\right|\right) + C$

(e) $\int \frac{\sin 2x}{1+\cos^4 x}\,dx =$ $-\tan^{-1}(\cos^2 x) + C$ (f) $\int x^2\cos x\,dx =$ $x^2\sin x + 2x\cos x - 2\sin x + C$

Bonus:

1. $\int \frac{x^2+5}{x^2+1}\,dx =$ $x + 4\tan^{-1}x + C$ (b) $\int \frac{1}{x^2+x-2}\,dx =$ $\frac{1}{3}\left(\ln|x-1| - \ln|x+2|\right) + C$

2. Write down the partial fractions decomposition of $\frac{3}{x^2(x^2+4)^2(x^2-16)}$. You may use $A, B, C, \ldots$ for the arbitrary constants. You need not find the values of the arbitrary constants.

$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2} + \frac{G}{x-4} + \frac{H}{x+4}$