Math 212 GH Quiz 5B
February 24, 2020

8 + 3 Bonus points possible!

Name: ANSWERS

Instructions: No calculators. Use your own scrap paper and write your answers in the space provided.

1. Suppose $\int_a^b f(x)\ dx$ is an integral in which $f(x)$ is a rational expression of trig functions.

   (a) State the traditional Weierstrass substitution: $u =$ $\tan\frac{x}{2}$

   (b) Using this substitution, derive or state $\sin x =$ $\frac{2u}{1+u^2}$ as a function of u

   (c) Using this substitution, derive or state $\cos x =$ $\frac{1-u^2}{1+u^2}$ as a function of u

   (d) Using this substitution, derive or state $dx =$ $\frac{2}{1+u^2}\,du$ in terms of du

   (e) Compute: $\int \frac{1}{\tan x + \sin x}\,dx =$ $\frac{1}{2}\left(\ln\left|\tan\frac{x}{2}\right| - \frac{\tan^2 x/2}{2}\right) + C$

2. Integrate the following:

   (a) $\int \frac{4x}{(1-4x)^3}\,dx =$ $\frac{1}{4}\left(\frac{1}{2(1-4x)^2} - \frac{1}{1-4x}\right) + C$

   (b) $\int \frac{x^2+7x+6}{x^2+x+3}\,dx =$ $x + 3\ln|x^2+x+3| + C$

   (c) $\int \frac{5}{x^2-x-6}\,dx =$ $\ln|x-3| - \ln|x+2| + C$ OR $\ln\left|\frac{x-3}{x+2}\right| + C$

Bonus:

1. Consider the integral $\int_a^b f(x)\ dx$. What three things will make the integral improper?

   (i) $f(x)$ is discontinuous at finitely many points in $[a,b]$

   (ii) $a = -\infty$

   (iii) $b = \infty$