

16 + 3 Bonus points possible

Name: ANSWERS

Instructions: No calculators. Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a)  $\int \sec^3 x \, dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$  (b)  $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

(c)  $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$  (d)  $\int \ln x \, dx = x \ln x - x + C$

(e)  $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$  (f)  $\int \tan x \, dx = \ln|\sec x| + C$  or  $-\ln|\cos x| + C$

2. Complete the following table of trig substitutions (the first row is an example):

Expression	Substitution	Identity
$a^2 + x^2$	$x = a \tan \theta$	$1 + \tan^2 \theta = \sec^2 \theta$
$x^2 - a^2$	$x = a \sec \theta$	$\sec^2 \theta - 1 = \tan^2 \theta$
$a^2 - x^2$	$x = a \sin \theta$	$1 - \sin^2 \theta = \cos^2 \theta$

3. Integrate the following:

(a)  $\int \sin^4 x \cos^3 x \, dx = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$  (b)  $\int \cos^2 x \, dx = \frac{1}{2}(x + \frac{1}{2} \sin 2x) + C$

(c)  $\int \theta \sec \theta \tan \theta \, d\theta = \theta \sec \theta - \ln|\sec \theta + \tan \theta| + C$  (d)  $\int \frac{x^2}{\sqrt{9-x^2}} \, dx = \frac{9}{2}(\sin^{-1}(\frac{x}{3}) - \frac{x\sqrt{9-x^2}}{9}) + C$

(e)  $\int t^2 \sin t \, dt = -t^2 \cos t + 2t \sin t + 2 \cos t + C$  (f)  $\int \arcsin x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C$

Bonus:

1.  $\int \frac{x^2 - 1}{x^2 + 1} \, dx = x - 2 \tan^{-1} x + C$  (b)  $\int \frac{1}{x^2 - x - 6} \, dx = \frac{1}{5}(\ln|x-3| - \ln|x+2|) + C$

2. Write down the partial fractions decomposition of  $\frac{12}{x^2(x^2+1)^2(x^2-9)}$ . You may use  $A, B, C, \dots$  for the arbitrary constants. You need not find the values of the arbitrary constants.

$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2} + \frac{G}{x-3} + \frac{H}{x+3}$