Math 212 GH Quiz 3A
February 10, 2020

(16) + 3 Bonus points possible.

Name: ANSWERS

Instructions: No calculators. Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a) $\int \ln x \, dx =$ $x\ln x - x + C$ (b) $\int \sec x \, dx =$ $\ln|\sec x + \tan x| + C$

(c) $\int \tan x \, dx =$ $\ln|\sec x| + C$ OR $-\ln|\cos x| + C$ (d) $\int \sec^3 x \, dx =$ $\frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$

(e) $\int \frac{1}{1+x^2} dx =$ $\tan^{-1} x + C$ (f) $\int \frac{1}{\sqrt{1-x^2}} dx =$ $\sin^{-1} x + C$

2. Complete the following table of trig substitutions (the first row is an example):

Expression	Substitution	Identity
$a^2 - x^2$	$x = a\sin\theta$ or $x = a\cos\theta$	$1 - \sin^2\theta = \cos^2\theta$ or $1 - \cos^2\theta = \sin^2\theta$
$a^2 + x^2$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$x^2 - a^2$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

3. Integrate the following:

(a) $\int \sin^5 x \cos^3 x \, dx =$ $\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C$ OR $-\frac{\cos^8 x}{8} + \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$ (b) $\int \sin^2 x \, dx =$ $\frac{1}{2}(x - \frac{1}{2}\sin 2x) + C$

(c) $\int \sec^2\theta \ln\tan\theta \, d\theta =$ $\tan\theta \ln\tan\theta - \tan\theta + C$ (d) $\int \frac{x^2}{\sqrt{x^2+9}} dx =$ $\frac{9}{2}\left(\frac{x\sqrt{x^2+9}}{9} - \ln|\sqrt{x^2+9} + x|\right) + C$

(e) $\int t^2 \cos t \, dt =$ $t^2\sin t + 2t\cos t - 2\sin t + C$ (f) $\int \arcsin x \, dx =$ $x\sin^{-1} x + \sqrt{1-x^2} + C$

Bonus:

1. $\int \frac{x^2-4}{x^2+1} dx =$ $x - 5\tan^{-1} x + C$ (b) $\int \frac{1}{x^2+5x+6} dx =$ $\ln|x+2| - \ln|x+3| + C$

2. Write down the partial fractions decomposition of $\frac{12}{x^2(x^2+4)^2(x^2-1)}$. You may use $A, B, C, \ldots$ for the arbitrary constants. You need not find the values of the arbitrary constants.

$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2} + \frac{G}{x-1} + \frac{H}{x+1}$