Math 205 Final Exam

Fall 2016

December 13, 2016

Name: ____

Info and Instructions:

1. Note that this exam is 2 hours and 15 minutes long.

2. **Complete all problems in part 1 and omit at most two from part 2—9 problems total**. In part 1, points are indicated. In part 2, each problem is worth 10 points.

3. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers. **If you need more space, use the blank page to the <u>LEFT</u> of the problem you're working on.**

4. Write neatly so that I am able to follow your sequence of steps, and box or otherwise indicate your answers where necessary.

5. Calculators are NOT permitted, no scrap paper, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting. In fact, **cell phones should be out of sight! You will be asked to leave the exam if you're caught with your phone or cheating (and you'll receive an F for the course in the latter case).**

6. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

7. Other than that, have fun and good luck!

<u>You may indicate what problems you wish to OMIT below</u>

Part 1: Do all			**Part 2: Omit two**	
Problem	**Points Earned**		**Problem**	**Points Earned**
1			6	
2			7	
3			8	
4			9	
5			10	
			11	
			Total	

Part 1: Do all problems in this part.

1. (4 points each part) Find $\frac{dy}{dx} = y'$ for each of the following:

(a) $y = e^{5x}\sqrt{4 - x^3}$

(b) $y = \frac{x^2 \ln x}{(1-x)^3}$

(c) $y = \sqrt[3]{x}(\ln x^5 + (\ln x)^5)$

(d) $y = \ln\left[\frac{\pi x^2 e^{3x}}{\sqrt{x}(x+2)^3(7-x)}\right]$

2. (5 points each) Evaluate the following integrals:

(a) $\displaystyle\int \frac{5}{3x\sqrt{\ln x}}\, dx$

$(b)\quad \int \frac{(3+x)(2-x^2)}{x^3}\,dx$

$(c)\quad \int \frac{4}{e^{5x}}\,dx$

$(d)\quad \int_0^1 \frac{2x}{\sqrt{x^2+1}}\,dx$

(e) $\displaystyle\int_{1}^{\sqrt[3]{\ln 3}} x^2 e^{x^3}\, dx$

3. (6 points) Find $\frac{dy}{dx}$ if $\sqrt{x+y} + e^{x+y} + xy^3 + 3y = 3 - 2x$.

4. The half-life of a substance is 21 days. Assume you start with 21 grams of the sample and let $P(t)$ be the amount of the substance remaining after t days.
   (a) (3 points) Find the differential equation satisfied by $P(t)$ and also its initial condition.

   (b) (2 points) Find $P(t)$ and simplify.

5. (a) (5 points) State the limit definition to find the derivative $f'(x)$ of a function $f(x)$ and use this definition to find $f'(3)$ for the function $f(x) = 3 - \frac{2}{1+x}$

   (b) (3 points) Use part (a) to find the tangent line to $y = f(x)$ at the point where $x = 1$.

Part 2: Complete any four problems in this section. Each problem is worth 10 points.

6. Sketch the function $f(x) = \frac{x}{1-x^2}$ by first finding (provided they exist) the domain, intercepts, asymptotes, local extrema, inflection point(s), intervals of increasing and decreasing, and intervals of concavity. You may assume, without verification, that $f'(x) = \frac{x^2+1}{(1-x^2)^2}$ and $f''(x) = \frac{2x(x^2+3)}{(1-x^2)^3}$.

7. (a) A particle is traveling on the curve $y^2 + xy = 2$. As the particle goes through the point (1,1), the x-coordinate is decreasing at a rate of 3 units per second. Find the rate at which the y-coordinate is changing at this moment.

(b) Compute the following limits:

$(i)\ \lim_{x \to 1} \frac{x^2 - 1}{1 - x^4}$

$(ii)\ \lim_{x \to -\infty} \frac{3 - x^{\pi} + 5x^{12}}{4 + 3x^2 - 2x^5 - x^{12}}$

8. A closed rectangular box with a square base and volume 12 cubic feet is to be constructed using two different types of materials. The top is made of metal costing \$2 per square foot, and the remaining sides and the base are made of wood costing \$1 per square foot. Find the dimensions of the box that minimizes the cost of construction.

9. (a) Use Riemann sums with 4 subintervals and right hand endpoints to estimate the area under $y = x^2$ on $0 \leq x \leq 1$. You may leave your answer as a sum of fractions.

(b) Use integration to find the exact area under the curve, which you estimated above.

10.

(a) An object is launched from a height of 96 feet at an initial velocity of 16 ft/sec. Find the position of the object, $s(t)$, that gives its height at time t, in seconds. For this problem, note that the acceleration due to gravity is -32 ft/sec^2.

(b) When will the object hit the ground?

(c) How high will the object get?

11. Roughly sketch the curves $y = -x^2 + 6x - 5$ and $y = 2x - 5$ on the same pair of axes, shade the area enclosed between them, and compute this area.