**Math 201 Test 2 Review 1**

December 5, 2019

**Name:** SOLUTIONS

**Note that both sides of each page may have printed material.**

**Instructions:**

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic**! Don't look down, while you're at it.

3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.

4. **You have 1 hour to complete the test**.

5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.

6. Write neatly so that I am able to follow your sequence of steps and box your answers.

7. Read through the exam and complete the problems that are easy (for you) first!

8. Scientific calculators are allowed, but you are NOT allowed to use notes, graphing calculators, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

9. In fact, **cell phones should be out of sight!**

10. Use the correct notation and write what you mean! $x^2$ and $x2$ are not the same thing, for example, and I will grade accordingly.

11. Other than that, have fun and good luck!

    May the force be with you. But you can't ask it to help you with your test.

1. Compute the following integrals (5 points each):

(a) $\int \frac{x^2 - 3x + 4}{x^2} dx$ $= \int 1 - \frac{3}{x} + \frac{4}{x^2} \, dx$

$= \boxed{x - 3\ln|x| - \frac{4}{x} + C}$

(b) $\int \frac{e^{\sin^{-1} x}}{\sqrt{1 - x^2}} dx$

$u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}} dx$

$\Rightarrow dx = \sqrt{1-x^2}\, du$

$\int \frac{e^u}{\cancel{\sqrt{1-x^2}}} \cdot \cancel{\sqrt{1-x^2}}\, du$

$= \int e^u du$

$= e^u + C$

$= \boxed{e^{\sin^{-1} x} + C}$

(c) $\int \tan x \ln(\cos x)\, dx$

$u = \ln(\cos x)$

$du = -\frac{\sin x}{\cos x} dx$

$\Rightarrow dx = -\frac{1}{\tan x} du$

$\int \cancel{\tan x} \cdot u \cdot -\frac{1}{\cancel{\tan x}} du$

$= -\int u \, du$

$= -\frac{u^2}{2} + C$

$= \boxed{-\frac{(\ln \cos x)^2}{2} + C}$

(d) $\int_0^{\pi/4} 2\cos^2 2x - 1\,dx = \int_0^{\pi/4} \cos 4x\,dx$ — double angle formula

$= \frac{1}{4}\sin 4x \Big|_0^{\pi/4}$

$= \frac{1}{4}\sin\pi - \frac{1}{4}\sin 0$

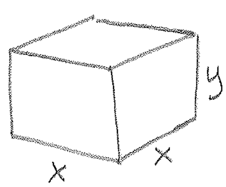
$= \boxed{0}$

(We could also notice that the period of $\cos 4x$ is $\pi/4$, and so "0" is immediate by a rule mentioned in class.)

2. (15 points) A storage shed is to be built in the shape of a box with a square base. It is to have a volume of 300 cubic feet. The base is concrete and costs \$8 per square foot, the roof is made of a metal that costs \$4 per square foot, and the material for the sides costs \$5 per square foot. Find the dimensions of the most economical shed.

① Read

② Diagram




③ Constraint

$x^2 y = 300$

Objective

$C = \underbrace{8x^2}_{\text{base}} + \underbrace{4x^2}_{\text{roof}} + \underbrace{20xy}_{\text{sides}}$

$C = 12x^2 + 20xy$

④ New Objective

$y = \frac{300}{x^2} \Rightarrow C = 12x^2 + 20x\left(\frac{300}{x^2}\right)$

$\Rightarrow C = 12x^2 + \frac{6000}{x}$

⑤ Minimize C

$C' = 24x - \frac{6000}{x^2}$

For crit pt: $x = 0 \rightarrow$ reject, or

$24x - \frac{6000}{x^2} = 0$

$\Rightarrow 24x^3 - 6000 = 0$

$\Rightarrow x^3 = \frac{6000}{24} = 250$

$\Rightarrow x = \sqrt[3]{250}$

$\Rightarrow x = 5\sqrt[3]{2}$

$\Rightarrow y = \frac{300}{25\sqrt[3]{2}^2} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$

$y = 6\sqrt[3]{2}$

⑥ Dimensions: $\boxed{5\sqrt[3]{2} \times 5\sqrt[3]{2} \times 6\sqrt[3]{2}}$

3. (a) (10 points) A moving particle has an acceleration of $a(t) = 3t^3$. If its initial velocity is $v(0) = 1$ and its initial position is $s(0) = 0$, find the position function of the particle.

Does the particle ever change direction?

$a(t) = 3t^3$

$\Rightarrow v(t) = \int 3t^3\,dt = \frac{3}{4}t^4 + C$

$v(0) = 1 \Rightarrow C = 1$

$\Rightarrow v(t) = \frac{3}{4}t^4 + 1$

$\Rightarrow s(t) = \int \frac{3}{4}t^4 + 1\,dt$

$= \frac{3}{20}t^5 + t + C$

$s(0) = 0 \Rightarrow C = 0$

$\Rightarrow \boxed{s(t) = \frac{3}{20}t^5 + t}$

Since $v(t) = \frac{3}{4}t^4 + 1 > 0$ for all $t$,

$\boxed{\text{the particle never changes direction.}}$

(b) (10 points) Compute $\frac{d}{dx}\int_x^{3x^3} \sqrt[3]{1+t^3}\,dt$ $= \frac{d}{dx}\left(\int_c^{3x^3} \sqrt[3]{1+t^3}\,dt - \int_c^{x} \sqrt[3]{1+t^3}\,dt\right)$

$= \sqrt[3]{1+(3x^3)^3} \cdot 9x^2 - \sqrt[3]{1+x^3}$

$= \boxed{9x^2\sqrt[3]{1+27x^9} - \sqrt[3]{1+x^3}}$

4. (5 points) Use linear approximation to approximate $\sqrt{35.5}$.

Set $f(x) = \sqrt{x}$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

Set $x = 35.5$ and $a = 36$.

Using $f(x) \approx f(a) + f'(a)(x-a)$

we have,

$\sqrt{35.5} = f(35.5) \approx f(36) + f'(36)(35.5-36)$

$= 6 + \frac{1}{12}\left(-\frac{1}{2}\right)$

$= 6 - \frac{1}{24} = \boxed{\frac{143}{24}}$

5. (a) (10 points) Find the value(s) of $c$ guaranteed by the Mean Value Theorem for the function $f(x) = x(x^2 - x - 2)$ on the interval $[-1,1]$.

$x^3 - x^2 - 2x$

What conditions allowed you to apply the Mean Value Theorem here?

We can apply the MVT since $f$ is continuous on $[-1,1]$ and differentiable on $(-1,1)$.

$$f_{avg} = \frac{f(1) - f(-1)}{1-(-1)} = \frac{-2-0}{2} = -1$$

$$f'(x) = 3x^2 - 2x - 2$$

By MVT, $\exists\, c \in (-1,1)$ s.t.

$$f'(c) = f_{avg}$$

$$\Rightarrow 3c^2 - 2c - 2 = -1$$

$$\Rightarrow 3c^2 - 2c - 1 = 0$$

$$\Rightarrow (3c+1)(c-1) = 0$$

$$\Rightarrow \boxed{c = -\frac{1}{3}} \quad \text{or} \quad \cancel{c = 1}$$

reject! Not in $(-1,1)$.

(b) (10 points) Find the absolute extrema of the function $f(x) = 3x^4 - 4x^3$ on $[-1,2]$.

1 Crit pts

$f' = 12x^3 - 12x^2 = 0$ for crit pt

$\Rightarrow 12x^2(x-1) = 0$

$\Rightarrow x = 0, x = 1$ are crit pts.

$f(0) = 0$

$f(1) = -1$

2 Endpoints

$f(-1) = 7$

$f(2) = 16$

3 Compare

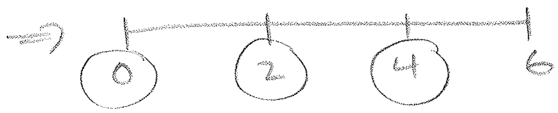
$f(2) = 16 \rightarrow$ abs max

$f(1) = -1 \rightarrow$ abs min

6. Consider the function $f(x) = x^2$ on the interval $[0,6]$.

(a) (10 points) Use a finite Riemann sum with three equal subintervals and left-hand endpoints to approximate the area under $f(x)$ on the interval.

$$\Delta x = \frac{b-a}{n} = \frac{6-0}{3} = 2$$




$$A \approx L_3 = \left(f(0) + f(2) + f(4)\right)\Delta x$$
$$= (0 + 2^2 + 4^2)(2)$$
$$= \boxed{40}$$

(b) (10 points) Compute the exact area under $f(x)$ on the interval by using the limit at infinity of a Riemann sum.

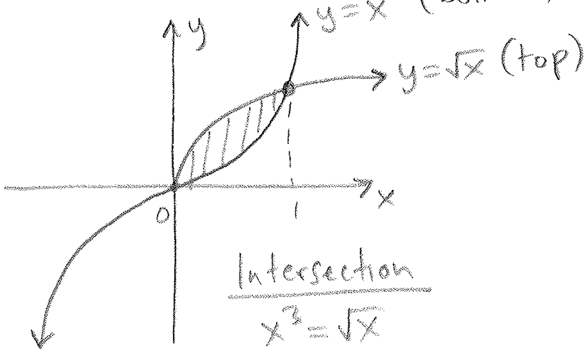
$$\Delta x = \frac{b-a}{n} = \frac{6}{n}$$

$$x_i = a + i\Delta x = \frac{6i}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6i}{n}\right)^2 \cdot \frac{6}{n}$$
$$= \lim_{n\to\infty} \frac{6^3}{n^3} \sum_{i=1}^{n} i^2$$
$$= \lim_{n\to\infty} \frac{6^{\cancel{3}\,2}}{n^3} \cdot \frac{n(n+1)(2n+1)}{\cancel{6}}$$
$$= \boxed{72}$$

**Bonus Problems:** 5 points each. (You must complete all problems in the actual test to be eligible).

1. Compute the area bounded between the curves $y = x^3$ and $y = \sqrt{x}$. Include a sketch of the bounded region.




Intersection

$x^3 = \sqrt{x}$

$\Rightarrow x=0, x=1.$

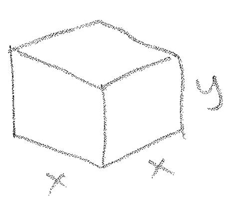
$$A = \int_0^1 \sqrt{x} - x^3\,dx$$

$$= \frac{2}{3}x^{3/2} - \frac{x^4}{4}\Big|_0^1$$

$$= \frac{2}{3} - \frac{1}{4}$$

$$= \boxed{\frac{5}{12}}$$

3. A box has a square base of side length 10cm and a height of 15cm. But I could be wrong. Assuming I could make an error of up to 0.1cm, use differentials to compute the maximum possible error in finding the volume of the box.




$dx = 0.1$

$dy = 0.1$

$$V = x^2 y$$

$$\Rightarrow dV = 2xy\,dx + x^2\,dy$$

$$= 2(10)(15)(0.1) + (10)^2(0.1)$$

$$= 30 + 10$$

$$= \boxed{40} \text{ cubic units, max error.}$$