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Math 201 Quiz 5A

September 23, 2019

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided.

1. Using an equation, define what it means for a function $f(x)$ to be *continuous* at $x = a$:

   $\lim_{x \to a} f(x) = f(a)$

2. What does it mean to say $f(x)$ is *continuous*?

   f is continuous at all points in its domain

3. Name the six kinds of *elementary continuous functions*: (may list in any order).

   Polynomials — Trig functions

   Rational functions — Exponential functions

   Radical functions — Logarithmic functions

4. State where the following are continuous. Use interval notation.

   (a) $f(x) = \ln(x^2 - 1)$ $(-\infty, -1) \cup (1, \infty)$ (b) $g(x) = \frac{3x+1}{\sqrt{5+x} - \sqrt{4-x}}$ $[-5, -\frac{1}{2}) \cup (-\frac{1}{2}, 4]$

5. Find the value(s) of a and b that make the function $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \\ ax^2 + bx + 2 & \text{if } 1 \le x < 2 \\ 2x - a + b & \text{if } x \ge 2 \end{cases}$ continuous.

   $a =$ 1/2 $\quad b =$ −1/2

   If there are no such a and b, state "N/A" for each of the above two slots.

Bonus:

1. Consider the function $f(x) = x - \cos x$

   (a) Compute $f(0) =$ −1 and $f(2) =$ $2 - \cos 2$

   (b) Does the equation $f(x) = 0$ have at least one solution? (Yes/no) Yes

   (c) How do you know? Apply the Intermediate Value Theorem to f.