Math 201 Mock Quiz 12 Answers

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Name: ANSWERS

Instructions: No calculators. Use provided scrap. Write your fully simplified answers in the space provided.

1. Complete the following anti-derivative formulas:

You will lose ½ pt if you forget "+C"

You will lose ½ pt if you forget the abs value signs.

$(a)\ \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ $\quad (b)\ \int \frac{1}{x}\,dx = \ln|x| + C$

$(c)\ \int e^x\,dx = e^x + C$ $\quad (d)\ \int \sin x\,dx = -\cos x + C$

$(e)\ \int \cos x\,dx = \sin x + C$

2. Find the general anti-derivatives of:

$(a)\ \int \sec^2 x + 9x^4\,dx = \tan x + \frac{9}{5}x^5 + C$ $\quad (b)\ \int 14x - 11\,dx = 7x^2 - 11x + C$

3. Find a function $f(x)$ that satisfies the conditions: $f''(1) = 4,\ f'(1) = -8,\ f(1) = 13.$

$f(x) = 2x^2 - 12x + 23$ OR other function w/ these properties.

4. A shipping company needs to design a closed rectangular shipping crate with a square base that can hold 45000 cubic feet of cargo. The material for the top and sides will cost \$3 per square foot, while the material for the base will cost \$7 per square foot. Calling the base length x and the height y,

(a) State the objective equation: $C = 10x^2 + 12xy$

(b) State the constraint equation: $x^2 y = 45000$

(c) What dimensions will minimize the cost of materials? $30 \times 30 \times 50$ ft
(State your answer in $length \times width \times height$ format.)

Bonus (Complete the other problems to be eligible):

1. Compute the following:

$(a)\ \sum_{n=0}^{4} \sin\frac{n\pi}{2} = 0$ $\quad (b)\ \sum_{i=1}^{n} (i+1)(i+2) = \frac{n}{3}(n^2 + 6n + 11)$

$(c)\ \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\left[\left(\frac{2i}{n}\right)^3 + 5\left(\frac{2i}{n}\right)\right]\right) = 14$

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The problems before this require direct application of formulas or memorization. Let's start with problem 3.

3/ Assume $f''(x) = 4$, a constant, to keep it simple.

$f''(1)=4$ is automatic.

$\Rightarrow f'(x) = \int 4\,dx = 4x + C$

since $f'(1) = -8 \Rightarrow 4 + C = -8 \Rightarrow C = -8 - 4 = -12$

$\Rightarrow f'(x) = 4x - 12$

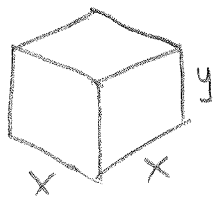
$\Rightarrow f(x) = \int 4x - 12\,dx = 2x^2 - 12x + C$

Since $f(1) = 13 \Rightarrow 2 - 12 + C = 13 \Rightarrow C = 23$

$\Rightarrow \boxed{f(x) = 2x^2 - 12x + 23} \rightarrow$ is such a function.

4/ ① Read

② Diagram



③ Constraint

$x^2 y = 45000$

Objective

$$C = \underbrace{3x^2}_{\text{top}} + \underbrace{3(4xy)}_{\text{4 sides}} + \underbrace{7x^2}_{\text{base}}$$

$$\Rightarrow C = 10x^2 + 12xy$$

④ $y = \dfrac{45000}{x^2}$

$$\Rightarrow C = 10x^2 + \frac{12(45000)}{x}$$

⑤ Min C

$$C' = 20x - \frac{12(45000)}{x^2} \overset{\text{for crit pt.}}{=} 0 \text{ or } \underbrace{\cancel{\text{und}}}_{\text{reject.}}$$

$$\Rightarrow 20x^3 - 12(45000) = 0$$

$$\Rightarrow x^3 = \frac{\overset{6}{\cancel{12}}(4500\cancel{0})}{\cancel{2}\cancel{0}}$$

$$\Rightarrow x^3 = 27000$$

$$\Rightarrow x = \sqrt[3]{27000} = 30$$

$$\Rightarrow y = \frac{45000}{30^2} = \frac{45000}{9000} = 50$$

$\Rightarrow$ ⑥ Dimensions: $\boxed{30 \times 30 \times 50}$ ft

Bonus

(a) $$\sum_{n=0}^{4} \sin\frac{n\pi}{2} = \sin 0 + \sin\frac{\pi}{2} + \sin\frac{2\pi}{2} + \sin\frac{3\pi}{2} + \sin\frac{4\pi}{2}$$

$$\xrightarrow{} 0 \quad 1 \quad 0 \quad -1 \quad 0$$

$$= \boxed{0}$$

(b) $$\sum_{i=1}^{n} (i+1)(i+2) = \sum_{i=1}^{n}(i^2+3i+2)$$

$$= \sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 2$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + 2n$$

$$= \frac{n}{6}\left[(n+1)(2n+1) + 9(n+1) + 12\right]$$

$$= \frac{n}{6}\left[2n^2+3n+1+9n+9+12\right]$$

$$= \frac{n}{6}\left[2n^2+12n+22\right]$$

$$= \frac{n}{6}\cdot 2(n^2+6n+11)$$

$$= \boxed{\frac{n}{3}(n^2+6n+11)}$$

(c) $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\left[\left(\frac{2i}{n}\right)^3 + 5\left(\frac{2i}{n}\right)\right]\right) = \lim_{n\to\infty}\left(\frac{16}{n^4}\sum_{i=1}^{n} i^3 + \frac{20}{n^2}\sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty}\left(\frac{16}{n^4}\cdot\left[\frac{n(n+1)}{2}\right]^2 + \frac{20}{n^2}\cdot\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{16n^4+\cdots}{4n^4+\cdots} + \frac{20n^2+\cdots}{2n^2+\cdots}\right)$$

$$= 4 + 10$$

$$= \boxed{14}$$