Math 201 Mock Quiz 11 Answers

Name: ANSWERS

Instructions: No calculators. Use provided scrap. Write your fully simplified answers in the space provided.

1. For the function $f(x) = \frac{x^3}{x^2-1}$, you are given (and need not verify) that

$$f'(x) = \frac{x^2(x^2-3)}{(x^2-1)^2} \text{ and } f''(x) = \frac{2x(x^2+3)}{(x^2-1)^3}$$

Find, if they exist:

(a) The domain of $f(x)$: $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$

(b) Its x-intercept(s): $(0,0)$ "$x=0$" allowed (c) Its y-intercept: $(0,0)$ "$y=0$" allowed.

(d) Its vertical asymptote(s): $x=-1$, $x=1$

(e) Its horizontal asymptote(s): None

(f) Intervals of: increase: $(-\infty,-\sqrt{3}) \cup (\sqrt{3},\infty)$ decrease: $(-\sqrt{3},-1) \cup (-1,0) \cup (0,1) \cup (1,\sqrt{3})$

(g) Local max point(s): $(-\sqrt{3}, -3\sqrt{3}/2)$ Local min point(s): $(\sqrt{3}, 3\sqrt{3}/2)$

(h) Intervals of concavity: C.U. on: $(-1,0) \cup (1,\infty)$

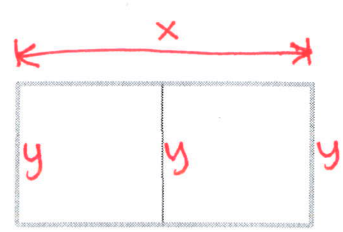
C.D. on: $(-\infty,-1) \cup (0,1)$

(i) Inflection point(s): $(0,0)$

Do your calculations on the provided scrap paper and sketch the graph of $f(x)$ on the reverse side of this page. Indicated the above features on your graph.

Bonus (Complete the other problems to be eligible):

1. A rectangular corral of 162 square-meters is to be fenced off and then divided by a fence into two sections, as shown in the figure to the right. Label this figure, using x for any horizontal dimensions and y for any vertical dimensions in your set-up.



If the amount of total fencing used is to be minimized, how much fencing is needed? $36\sqrt{3}$ meters

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① **Domain:** $\boxed{(-\infty,-1)\cup(-1,1)\cup(1,\infty)}$

② **Intercepts:** x-int: $0=\dfrac{x^3}{x^2-1} \Rightarrow \boxed{x=0}$

y-int: $y=\dfrac{0}{0-1} \Rightarrow \boxed{y=0}$

③ **Asymptotes:**

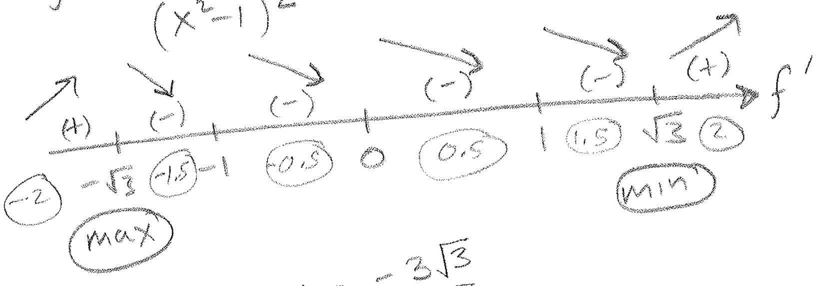
V.A.: $\boxed{x=\pm 1}$

H.A.: $\displaystyle\lim_{x\to\infty}\frac{x^3}{x^2-1}=\infty$

$\Rightarrow \boxed{\text{No H.A.}}$

④ **Inc/Dec/Max/Min**

$f'=\dfrac{x^2(x^2-3)}{(x^2-1)^2} \Rightarrow$ crit pts: $x=\pm1, 0, \pm\sqrt{3}$



when $x=-\sqrt{3}$, $y=-\dfrac{3\sqrt{3}}{2}$

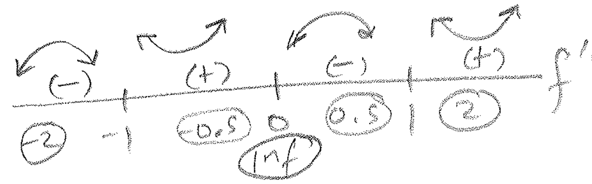
when $x=\sqrt{3}$, $y=\dfrac{3\sqrt{3}}{2}$

$\Rightarrow$

$$\boxed{\begin{array}{l}\text{Inc: } (-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)\\ \text{Dec: } (-\sqrt{3},-1)\cup(-1,0)\cup(0,1)\cup(1,\sqrt{3})\\ \text{Max: } (-\sqrt{3}, -3\sqrt{3}/2)\\ \text{min: } (\sqrt{3}, 3\sqrt{3}/2)\end{array}}$$

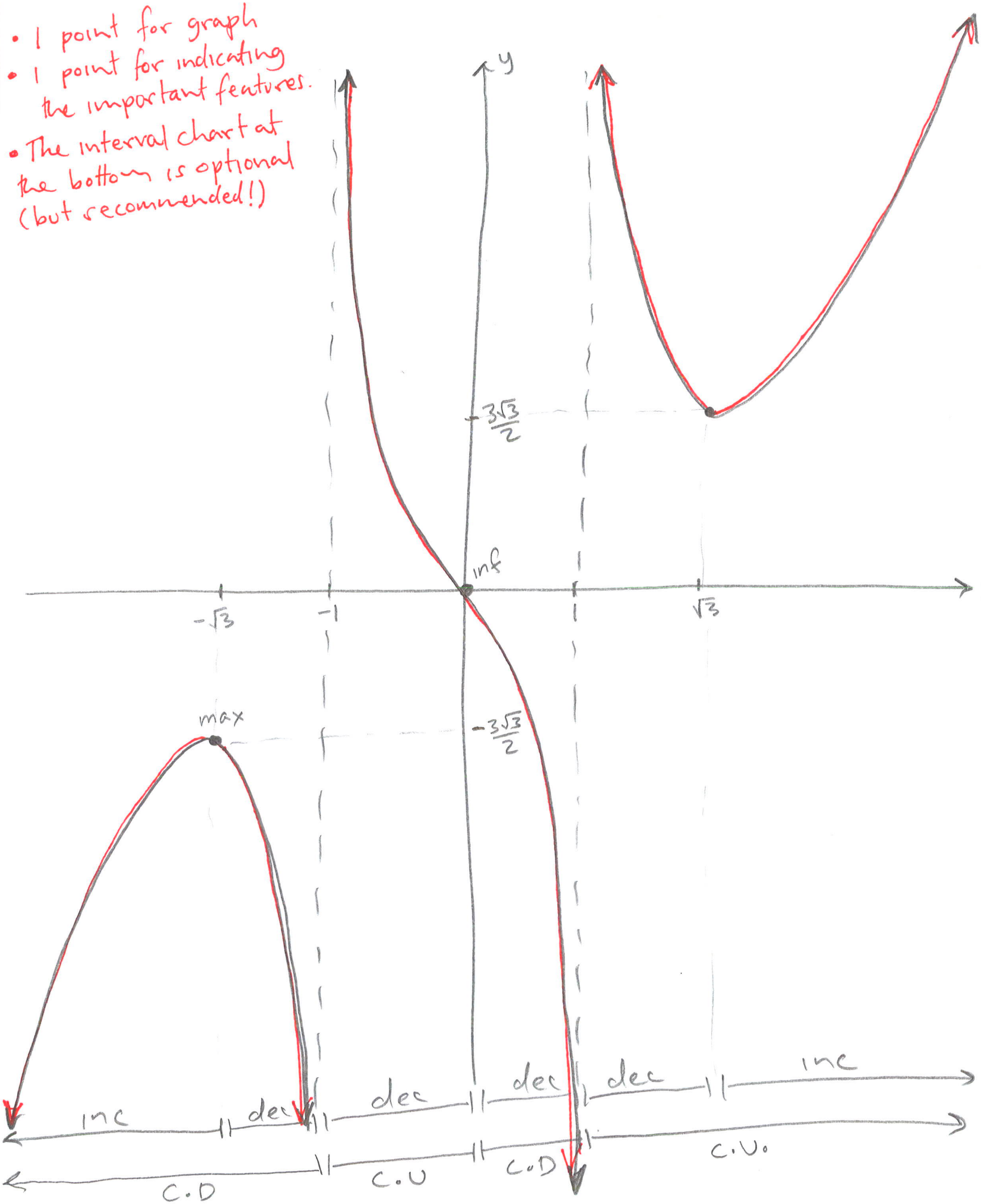
⑤ **C.U./C.D./Inf**

$f''=\dfrac{2x(x^2+3)}{(x^2-1)^3} \Rightarrow$ crit. pt: $x=\pm1, 0$



$$\boxed{\begin{array}{l}\text{C.U.: } (-1,0)\cup(1,\infty)\\ \text{C.D.: } (-\infty,-1)\cup(0,1)\\ \text{Inf: } (0,0)\end{array}}$$

- 1 point for graph
- 1 point for indicating the important features.
- The interval chart at the bottom is optional (but recommended!)



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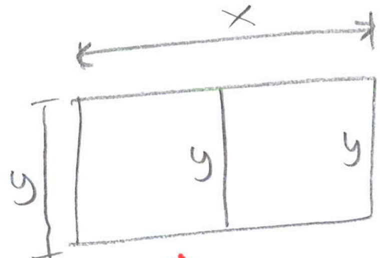
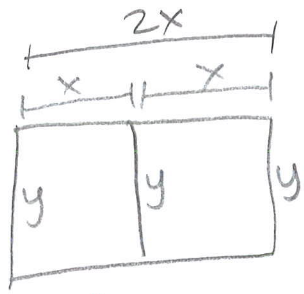
Bonus

① Read!

② Diagram



OR



I'll use this one!

③ Constraint

$xy = 162$

Objective

$P = 2x + 3y$

④ $x = \frac{162}{y}$

$\Rightarrow P = 2\left(\frac{162}{y}\right) + 3y$

$\Rightarrow P = \frac{324}{y} + 3y$

⑤ To find min P, set $P' = 0$ or und.

reject!

$P' = 0 \Rightarrow -\frac{324}{y^2} + 3 = 0$

$\Rightarrow 3y^2 = 324$

$\Rightarrow y^2 = 108$

$\Rightarrow y = +\sqrt{108} = \sqrt{36 \cdot 3}$

$= 6\sqrt{3}$

⑥ Amount of fencing

is $P = \frac{324}{6\sqrt{3}} + 3 \cdot 6\sqrt{3}$

$= \boxed{\frac{54}{\sqrt{3}} + 18\sqrt{3}}$ meters

OR

$= 18\sqrt{3} + 18\sqrt{3}$

$= \boxed{36\sqrt{3}}$