TEST 4 REVIEW

1/ $f(x) = 4+2x-x^2$

$$\Rightarrow \frac{f(x+h)-f(x)}{h}$$

$$= \frac{4+2(x+h)-(x+h)^2-(4+2x-x^2)}{h}$$

$$= \frac{\cancel{4}+\cancel{2x}+2h-\cancel{x^2}-2xh-h^2-\cancel{4}-\cancel{2x}+\cancel{x^2}}{h}$$

$$= \frac{2\cancel{h}-2x\cancel{h}-h^{\cancel{2}}}{\cancel{h}}$$

$$= 2-2x-h$$

$$\Rightarrow \frac{f(x+h)-f(x-h)}{h}$$

$$= \frac{4+2x+2h-x^2-2xh-h^2-(4+2(x-h)-(x-h)^2)}{h}$$

$$= \frac{\cancel{4}+\cancel{2x}+2h-\cancel{x^2}-2xh-\cancel{h^2}-(\cancel{4}+\cancel{2x}-2h-\cancel{x^2}+2xh-\cancel{h^2})}{h}$$

$$= \frac{4\cancel{h}-4x\cancel{h}}{\cancel{h}}$$

$$= 4-4x$$

2/ $f(x) = -x-2x^2$

$$\Rightarrow \frac{f(x+h)-f(x)}{h}$$

$$= \frac{-(x+h)-2(x+h)^2-(-x-2x^2)}{h}$$

$$= \frac{-\cancel{x}-h-\cancel{2x^2}-4xh-2h^2+\cancel{x}+\cancel{2x^2}}{h}$$

$$= \frac{-\cancel{h}-4x\cancel{h}-2h^{\cancel{2}}}{\cancel{h}}$$

$$= -1-4x-2h$$

2/ cont'd

$$\Rightarrow \frac{f(x+h)-f(x-h)}{h}$$

$$= \frac{-(x+h)-2(x+h)^2-(-(x-h)-2(x-h)^2)}{h}$$

$$= \frac{-\cancel{x}-h-\cancel{2x^2}-4xh-\cancel{2h^2}-(-\cancel{x}+h-\cancel{2x^2}+4xh-\cancel{2h^2})}{h}$$

$$= \frac{-2\cancel{h}-8x\cancel{h}}{\cancel{h}}$$

$$= -2-8x$$

3/ $f(x) = 25-10x+x^2$

evaluate the function as is, or note that

$$f(x) = (x-5)^2$$

(a) $f(-5) = (-10)^2 = 100$

(b) $f(5-a) - f(2a)$

$= a^2-(2a-5)^2$

$= a^2-4a^2+20a-25$

$= -3a^2+20a-25$

$= -(3a-5)(a-5)$

4/ $f(x) = \frac{1}{x+1}$

(a) $f(4a+2) = \frac{1}{4a+2+1} = \frac{1}{4a+3}$

(b) $f(1) = \frac{1}{1+1} = \frac{1}{2}$, $f(2a) = \frac{1}{2a+1}$

$\Rightarrow f(f(1)) - f(2a)$

$= f(\frac{1}{2}) - f(2a)$

$= \frac{1}{\frac{1}{2}+1} - \frac{1}{2a+1} = \frac{4a-1}{6a+3}$

TEST 4 REVIEW cont'd

5/ $h(x) = 6 - x^2,\ g(x) = \sqrt{7-x}$

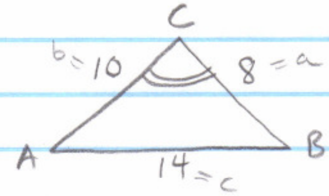
$$\Rightarrow h(g(x)) = 6 - (g(x))^2$$
$$= 6 - (\sqrt{7-x})^2$$
$$= 6 - (7-x)$$
$$= x - 1$$

$$\Rightarrow g(h(x)) = \sqrt{7 - (h(x))}$$
$$= \sqrt{7-(6-x^2)}$$
$$= \sqrt{x^2+1}$$

6/ $h(x) = \sqrt{\frac{2x+1}{3}},\ g(x) = \frac{3x^2-1}{2}$

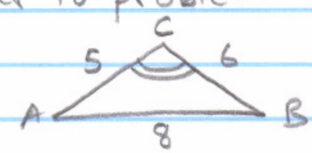
$$\Rightarrow h(g(x)) = \sqrt{\frac{2(g(x))+1}{3}}$$
$$= \sqrt{\frac{\cancel{2}\cdot\frac{3x^2-1}{\cancel{2}}+1}{3}}$$
$$= \sqrt{\frac{3x^2-\cancel{1}+\cancel{1}}{3}}$$
$$= \sqrt{\frac{\cancel{3}x^2}{\cancel{3}}}$$
$$= \sqrt{x^2} \rightarrow \text{technically } |x|$$
$$= x$$

$$g(h(x)) = \frac{3(h(x))^2 - 1}{2}$$
$$= \frac{\cancel{3}\left(\sqrt{\frac{2x+1}{\cancel{3}}}\right)^{\cancel{2}} - 1}{2}$$
$$= \frac{2x+\cancel{1}-\cancel{1}}{2}$$
$$= x$$

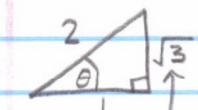
7/ 

The largest angle is C since it is opposite the longest side.
By the law of cosines:
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\Rightarrow C = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right)$$
$$= \cos^{-1}\left(\frac{8^2+10^2-14^2}{2(8)(10)}\right)$$
$$\approx 101.5^\circ$$

8/ Similar to problem 7.



$$\text{Largest angle} = C$$
$$= \cos^{-1}\left(\frac{5^2+6^2-8^2}{2(5)(6)}\right)$$
$$\approx 92.9^\circ$$

9/ $\sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow$ 

By Pythagoras

$$\Rightarrow \boxed{\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}}$$

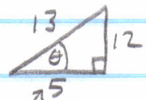
Also... $\sin\theta = \frac{\sqrt{3}}{2} \rightarrow \frac{\text{opp}}{\text{hyp}}$

$\csc\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \rightarrow \frac{1}{\sin\theta}$

$\tan\theta = \sqrt{3} \longrightarrow \frac{1}{\cot\theta}$

TEST 4 REVIEW cont'd

10/ $\csc\theta = \frac{13}{12} \Rightarrow \sin\theta = \frac{12}{13} \Rightarrow$



By Pythagoras

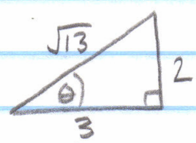
$\Rightarrow \boxed{\tan\theta = \frac{12}{5} \to \frac{\text{opp}}{\text{adj}}}$

Also... $\cos\theta = \frac{5}{13} \to \frac{\text{adj}}{\text{opp}}$

$\sec\theta = \frac{13}{5} \to \frac{1}{\cos\theta}$

$\cot\theta = \frac{5}{12} \to \frac{1}{\tan\theta}$

11/ $\sin\theta = \frac{2}{\sqrt{13}} = \frac{\text{opp}}{\text{hyp}} \Rightarrow$



By Pythagoras

$\Rightarrow \csc\theta = \frac{\sqrt{13}}{2} \to \frac{1}{\sin\theta}$

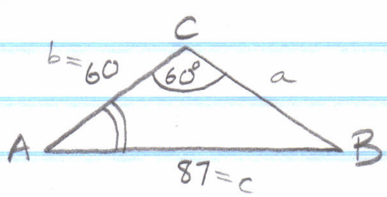
$\Rightarrow \cos\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13} \to \frac{\text{adj}}{\text{hyp}}$

$\Rightarrow \sec\theta = \frac{\sqrt{13}}{3} \to \frac{1}{\cos\theta}$

$\Rightarrow \tan\theta = \frac{2}{3} \longrightarrow \frac{\text{opp}}{\text{adj}}$

$\Rightarrow \cot\theta = \frac{3}{2} \to \frac{1}{\tan\theta}$

12/



By the law of sines

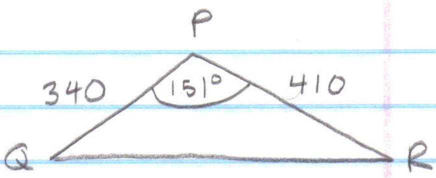
$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow B = \sin^{-1}\left(\frac{b\sin C}{c}\right) \approx 36.7°$

$\Rightarrow A = 180 - B - C$

$\therefore \boxed{A = 83.3°}$

13/



Let $PQ = r$, $PR = q$, $QR = p$

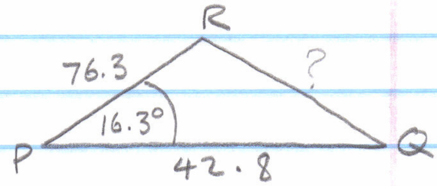
By the law of cosines

$p^2 = q^2 + r^2 - 2qr\cos P$

$\Rightarrow QR = p = \sqrt{410^2 + 340^2 - 2(410)(340)\cos 151°}$

≈ 726.32 meters.

14/



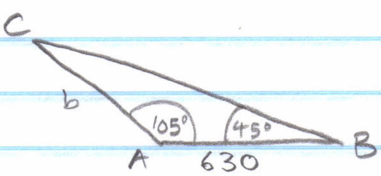
Let $PQ = r$, $PR = q$, $QR = p$

Similar to problem 13/

$QR = p = \sqrt{76.3^2 + 42.8^2 - 2(76.3)(42.8)\cos 16.3}$

≈ 37.21 meters.

15/



Note $C = 180° - 105° - 45° = 30°$

Then, by the law of sines

$\frac{b}{\sin B} = \frac{c}{\sin C}$

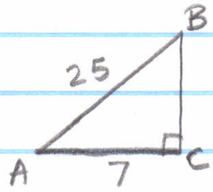
$\Rightarrow b = AC = \frac{c\sin B}{\sin C}$

$= \frac{630\sin 45}{\sin 30}$

≈ 890.95

Test 4 Review cont'd

16/



By Pythagoras

$25^2 = 7^2 + BC^2$

$\Rightarrow BC^2 = 25^2 - 7^2$

$= 576$

$\Rightarrow BC = 24$

17/ $\sin A = \frac{24}{25}$

$\Rightarrow A = \sin^{-1}\left(\frac{24}{25}\right)$

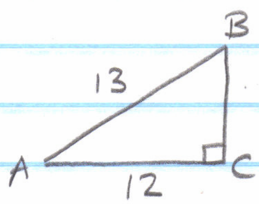
$\approx 73.7°$

$\Rightarrow B = 180° - 90° - 73.7°$

$= 16.3°$

I used 1 d.p. you can use whatever you want since the problem never told you how many d.p. to put.

18/



By Pythagoras

$13^2 = 12^2 + BC^2$

$\Rightarrow BC^2 = 13^2 - 12^2$

$= 25$

$\Rightarrow BC = 5$

19/ $\sin A = \frac{5}{13}$

$\Rightarrow A = \sin^{-1}\left(\frac{5}{13}\right)$

$\approx 22.6°$

$\Rightarrow B = 180° - 90° - 22.6°$

$= 67.4°$