Math 231 Take Home Test 3
Due June 4, 2015

Name: _____

Info and Instructions:

1. Complete all problems in the space provided. In this exam, each problem is worth 10 points, for a total of 50 points.

2. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, give an example, etc.)

3. Write neatly and logically so that I am able to follow your sequence of steps, box or otherwise indicate your answers where necessary.

4. No outside sources, you should be able to do all problems based on your class notes. I'm trusting that you will abide by this rule :p

5. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

6. Use the conventions we've used in class to express your answers. Being able to explain your reasoning in a clear way is a very important part of this class. Any one of your classmates should be able to read your solution and understand it. (Note: solutions should NOT be shared before the test is collected, this is just an example of the level of detail I want you to use. Do not skip steps that someone with only a background of calculus 1 would miss.)

7. Under no circumstances will this test be accepted late. You can place your test in my mailbox in the math dept, I will pick them up from there.

8. I require physical copies of your completed test. Sending me scanned or electronic copies is not recommended. However, if you must send such, be sure that I receive it before 10:30am on June 4, 2015. And be sure that the test is scanned into a pdf document that will be legible when printed. No other formats are acceptable. I should be able to just open your file and press print with no trouble, and read the printout with no trouble. In any case, your solutions should be hand-written.

9. Your name should be on your test and it should be stapled. I will deduct 5 points otherwise.

10. Other than that, have fun and good luck!

1. Solve the following recurrence relations:
   (a) $a_n = 6a_{n-1} - 8a_{n-2}, n \geq 2, a_0 = 4, a_1 = 10$

   (b) $a_{n+2} = -4a_{n+1} + 5a_n, n \geq 0, a_0 = 2, a_1 = 8$

   (c) $a_n = -a_{n-2}, n \geq 2, a_0 = 2i, a_1 = 0$

   (d) $a_n + 6a_{n-1} = -9a_{n-2}, n \geq 2, a_0 = 3, a_1 = 0$

2. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 1000 elements in A_2 and 10,000 elements in A_3 if:

(a) $A_1 \subseteq A_2$ and $A_2 \subseteq A_3$

(b) The sets are pairwise disjoint

(c) There are two elements in common to each pair of sets, and one element in common for all three sets.

3. Find the number of solutions to $x_1 + x_2 + x_3 + x_4 = 17$, where $x_i, i = 1,2,3,4$, are nonnegative integers with $x_1 \leq 3, x_2 \leq 4, x_3 \leq 5, x_4 \leq 8$. Hint: see example 1 in section 8.6 of the text.

4. (a) How many onto functions are there from a set with seven elements to one with five elements. Hint: you may or may not use the method of example 2 in section 8.6.

(b) List all the derangements of {1,2,3,4}. Use a formula to check that you have the correct number of them.

5. Let $A = \{a, b, c, d\}$ and let $R = \{(a,a), (a,b), (a,c), (a,d), (b,b), (b,c), (b,d), (c,c), (c,d), (d,d)\}$ be a relation on A. Which of the properties reflexive, symmetric, antisymmetric and transitive does R posses? Justify your answers.

Bonus Problems:

6. Let R be a relation defined on $\mathbb{Z}$ by aRb if $a + b$ is even. Show that R is an equivalence relation and determine the distinct equivalence classes.

7. In problem 6, suppose "even" is replaced by "odd". Which of the properties reflexive, symmetric and transitive does R posses?

8. For the set {1,2,3,4,5,6}, draw the Hasse diagram for:
   (a) Divisibility

   (b) "Greater than or equal to"