MATHE ??? FUNCTIONS
NOTES
SPRING 2012

MATTHEW AUTH, LECTURER OF MATHEMATICS

1. Introduction and Linear Functions (Week 1)

In this first week we give a preliminary definition of a function and learn the most basic method to draw its graph: First make a table of (x, y) values, then plot these points in the coordinate plane, and finally connect the points—to the best of your artistic and mathematical abilities.

The table method is tedious work. However, if you are persistent enough, it always results in a graph. Computers use the table method to graph functions. You should use it to graph a few of the assigned homework problems. After a couple exercises you will become bored with the table method and you will undoubtedly start looking for shortcuts. This is encouraged. We will spend a lot of time developing systematic methods to avoid making tables to graph functions.

One method to avoid making a table is roughly analogous to the way grade school students learn to multiply. In order to multiply any two numbers, no matter how large, students are only required to memorize how to multiply any two small numbers, numbers less than 10 in this case. They then learn the multiplicative algorithm which allows them to carry out any multiplication by breaking it down into potentially many multiplications of small numbers. For example

$$\begin{array}{ccccc} & & 3 & 8 & 4 \\ \times & & & 5 & 6 \\ \hline & 2 & 3 & 0 & 4 \\ 1 & 9 & 2 & 0 & \\ \hline 2 & 1 & 5 & 0 & 4. \end{array}$$

Things are not so tidy when graphing functions but we will memorize some basic graphs all the same and then break more complicated graphs into these basic ones, whenever possible. The most basic family of graphs is family of linear graphs like $f(x) = mx + b$. This is where we will start.

Next week we learn to graph functions like $g(x) = x^2$, the quadratic functions. We will then learn how to graph combinations of quadratic functions and linear functions. For example we learn to graph their sum $g(x) + f(x)$ and their composition $g(f(x)$. Only memorize a few basic graphs. Learn to combine them into more complicated graphs.

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2. Quadratic Functions (Week 2)

The linear functions are part of a larger family, the polynomial functions. Amongst themselves polynomials are distinguished by their degree. The polynomial functions of degree one are the linear functions. This week we will study the graphs of polynomial functions of degree two, the quadratic functions. Degree two polynomials are not much harder to graph than lines but, like lines, they are important in applications. Moreover, the method we will use to graph quadratics, the transformational method, will be used repeatedly throughout the semester. It is a great way to avoid making tables. You are advised to learn it now.

As the semester progresses we will try to classify functions for the same reason that we tried to classify numbers in middle school—some numbers are integers, some are fractions, ... Classifying a large set into subsets, even roughly, often leads to better understanding. Having a panoramic view of the many varieties of functions is invaluable when trying to teach functions. We have already started to classify the polynomial family as well as its linear and the quadratic subfamilies.

There are more classes of functions at first. To the polynomial functions, we have to add the classes of rational functions, exponential functions, and trigonometric functions. However, as we found when trying to classify the numbers in middle school, there are never enough classes.

The number π, for example, is not rational. It requires a new class, the irrational numbers. In this case at least things are not as bad as they seem. Irrational numbers can be approximated to any degree of accuracy by rational numbers. For instance,

$$\pi \approx 3 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} + \frac{5}{10000}.$$

Surprisingly, as we will see at the end of the semester, functions behave similarly, more or less. There will be functions that do not fit into any of our classes but they can approximated to any degree of accuracy by functions we know. For example the function

$$g(x) = x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} + \dots$$

is not in any of our classes but it is a function nonetheless. We cannot graph it using this expression but we can approximate its graph using

$$g(x) \approx x - \frac{x^3}{3} + \frac{x^5}{10}.$$

Department of Mathematics, CCNY, NAC 6-288, New York, NY 10031
E-mail address: `mauth@ccny.cuny.edu`