# MATHE 4800C FOUNDATIONS OF ALGEBRA AND GEOMETRY
# WEEKLY HW
# FALL 2011

MATTHEW AUTH, LECTURER OF MATHEMATICS

## 1. Assignment (due Wednesday 21 September)

**Exercise 1**
Problems from Kiselev's Planimetry: 3, 11, 62, 67, 68.

## 2. Assignment (due Wednesday 21 September)

**Exercise 1**
Problems from Courant: p.8 Ex 1–4 (You need not fill in all of the multiplication tables—just fill in a couple entries of each to get the idea)
**Exercise 2**

(1) Write 100101001 (binary) in decimal.
(2) Write 49 (decimal) in the binary (dyadic) system.
(3) Make the following computation in binary: $10001 + 11$
(4) Make the following computation in binary: $111 + 1111$
(5) Make the following computation in binary: $11111 - 1001$
(6) Make the following computation in binary: $1011 \cdot 11$
(7) Make the following computation in binary: $111 \cdot 10101$
(8) Make the following computation in binary: $1101/101$.

## 3. Assignment (due Wednesday 21 September)

**Exercise 1**
Problems from Kiselev's Planimetry: 76, 86, 87, 96, 106-108, 117-119, 124, 125, 130.

## 4. Assignment (due Wednesday 5 October)

**Exercise 1**
Problems from Kiselev's Planimetry: 141, 144, 149, 154, 158, 161, 165, 182, 186, 200, 202.

## 5. Assignment (due Wednesday 5 October)

**Exercise 1**
Suppose a student who is also a baseball fan does not agree with the definition of addition of fractions

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

*Date*: November 21, 2011.

given on p.53 of Courant's text. She claims that the definition should read

$$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$$

because in a double-header over the weekend she got three hits in five at bats in the first game and one hit in four at bats in the second game giving her a total of four hits in nine at bats or

$$\frac{3}{5} + \frac{1}{4} = \frac{3+1}{5+4} = \frac{4}{9}.$$

Explain the flaw in her reasoning.

**Exercise 2**

In your own words state why $(-1) \cdot (-1) = 1$.

**Exercise 3**

Explain why mathematicians generalized from the natural numbers $\{0, 1, 2, \ldots\}$ to the rational numbers. Give a few reasons why students may feel an amount of doubt and trepidation when they encounter this generalization for the first time.

**Exercise 4**

Explain the statement, "The system of rational points, although it is everywhere dense, does not cover all of the number axis."

**Exercise 5**

Do Exercises 1—3 on p.60–61 in Courant. (Once you get the hang of the technique, you can skip some of the problems)

**Exercise 6**

What goes wrong if you try to use the technique from the previous exercise to show that $\sqrt{25}$ is irrational.

**Exercise 7**

Do exercise 332 in Kiselev

**Exercise 8**

Do exercises on p. 63 and pp. 66–68 in Courant

**Exercise 9**

Use the laws of arithmetic from p. 2 in Courant to explain how the multiplication and division algorithm work.

## 6. Assignment (due Wednesday 12 October)

**Exercise 1**

Problems from Kiselev's Planimetry: 239—241, 243, 244, 246, 250, 251, 254, 255, 265—269.

## 7. Assignment (due Wednesday 19 October)

**Exercise 1**

Problems from Kiselev's Planimetry: 349, 354, 357, 371, 373, 376, 384, 386, 387.

## 8. Assignment (due Wednesday 2 November—week 8 on schedule)

**Exercise 1**

Problems from Kiselev's Planimetry: 428–435, 440.

**Exercise 2**

(1) Prove that a triangle with sides $5, 12$, and $13$ units is a right triangle.

(2) One leg of a right triangle has length 1 units. The hypotenuse has length 3 units. Find the length of the other leg.
(3) Given two points in the plane A and B. Describe the geometric locus of points $X$ so that $AX^2 + BX^2 = AB^2$.
(4) Show that $\sin 29° = \cos 61°$.
(5) Verify the law of sines in a $30 - 60 - 90$ triangle.
(6) Find $\sin 75°$.
(7) Find the legs of a right triangle with hypotenuse 10 units and one acute angle $72°$.
(8) Given a triangle $ABC$. Show that $4AM^2 = 2AB^2 + 2AC^2 - BC^2$ when $AM$ is a median.

## 9. Assignment (due Wednesday 16 November—week 9 on schedule)

**Exercise 1**

Problems from Kiselev's Planimetry: 455, 460, 462, 465. nn

## 10. Assignment (due Monday 28 November—week 10 on schedule)

**Exercise 1**

(1) Exercise p. 91 Courant
(2) Exercises p. 97 Courant
(3) Find the center and the radius of the circle defined by the locus of points $P(x, y)$ that satisfy the equation $x^2 - 4x + y^2 + 2y - 6 = 0$.
(4) Prove that the four points $O(0, 0), A(1, 2), B(3, 1)$, and $A+B$ are the vertices of a parallelogram.
(5) Find an equation of the unique line passing through the points $P(1, -3)$ and $Q(-2, 7)$. What does it mean geometrically that the algebraic equation is the equation of the line? Is it the only equation or are there more such equations?
(6) Find the points of intersection of the circle from exercise 3 and the line from exercise 5.
(7) Draw the image of the figure whose shape is the letter "R" under a) reflection through a point, b) reflection by a line, c) rotation by $90°$ about the origin, d) translation by $OP$ when $P$ is the point $(1, 2)$.
(8) In the previous problem what ultimately happens if you a) first reflect by a point and then rotate by $90°$ about the origin, b) first reflect by a line and then rotate $45°$ about the origin.
(9) Give an example of two isometries (superpositions) $H$ and $G$ so that $H \circ G \neq G \circ H$.
(10) Find the smallest positive integer $k$ so that $F^k = I$ when a) $F$ is rotation by $30°$ about the origin, b) $F$ is reflection by the y-axis, c) $F$ is translation by $OP$ when $P$ is the point $(-1, 0)$.
(11) Let $R_{60°}$ be rotation by $60°$ about the origin and $H$ by the mirror reflection by the x-axis. Find a positive integer $k$ so that $HR_{60°} = R_{60°}^k H$.
(12) Describe the isometry $R_{30°}T_{(1,0)}$.
(13) Describe the isometry $FG$ when $F$ is rotation by $45°$ about the point $(0, 1)$ and $G$ is rotation by $90°$ about the origin a) geometrically, b) as a function of the plane.

(14) Describe the isometry $FG$ when $F$ is rotation by $120°$ about the point $(-1, 0)$ and $G$ is rotation by $240°$ about the point $(0, 1)$ a) geometrically, b) as a function of the plane.

(15) In class we showed that every rotation and reflection is the composition of two mirror reflections. Is this decomposition unique? (In other words is there only one way to decompose a rotation $R = m_1 \circ m_2$ in the same way there is only one way to decompose the integer 6 as a product of primes $6 = 3 \cdot 2$. We say that the integer 6 can be uniquely factored into primes. Can we say that a rotation can be uniquely factored into reflections?)

(16) In class we showed that the matrix $P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ gives projection onto the x-axis. Describe geometrically the map of the plane $PR_{45°}$.

(17) Can you represent the map from the previous problem using complex numbers?

(18) Find the matrix for projection on the line $y = x$.

## 11. Assignment (due Wednesday 7 December—week 11 on schedule)

**Exercise 1**

(1) Read pages 165–193 Courant.
(2) Do exercises in Courant p. 172, p. 174, p. 179, p. 185, p. 188.

## 12. Assignment (due Monday 12 December—week 12 on schedule)

**Exercise 1**

(1) Read pages 212–227 and pages 235–244 Courant.
(2) In your own words follow the argument on page 240 to describe why there are only five regular polyhedra.

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