Department of Mathematics, The City College of CUNY
Math 201 Final Exam – December 18, 2017

Instructions:

- Show all work inside this booklet.
- Calculators and all other electronic devices must be turned off and out of sight.
- Answer all questions in Part I (70 points) and 3 questions (10 points each) from Part II.
- When you finish the exam, cross out from the second row in the chart below the two problems from Part II that you have omitted. If you do not do this, the first three questions that you answer (even partially) will be graded. Otherwise do not write anything in the chart below.

Name: ________________________ **Instructor:** ____________

Part I	[**1**]10	[**2**]10	[**3**]10	[**4**]10	[**5**]10	[**6**]10	[**7**]10	**Part I total**
Answer all problems								

Please leave these boxes blank!

Part II	[**8**]10	[**9**]10	[**10**]10	[**11**]10	[**12**]10	**Part II total**
Answer 3 out of 5 problems						

Cross out the problems you have omitted!

TOTAL: __________

Part I, questions 1 to 7. Answer all questions.

[1] **(10 pts)** Compute $f'(x)$ for each of the functions below. You do not need to simplify your answer.

(a) $f(x) = \tan\left(x\sqrt{1+x^2}\right).$

(b) $f(x) = \sin(x^3)\cos^2(5x)$

(c) $f(x) = \left(\dfrac{3x-4}{x^2+7}\right)^7$

[1] (10 pts)

Please leave blank!

[2] (10 pts) Find each integral:

(a) $\displaystyle\int (x^2 - 1)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

(b) $\displaystyle\int_0^1 \frac{x^{99}}{(7x^{100} + 1)^3}\, dx$

(c) $\displaystyle\int \frac{\sin(x)}{\cos^2(x)}\, dx$

[**2**] (10 pts)

Please leave blank!

[3] (10 pts) Find the limits, or state that the limit does not exist (you must justify your answer):

(a) $\displaystyle\lim_{x\to -1} \frac{x^2+3x+2}{x^2-3x-4}$

(b) $\displaystyle\lim_{x\to +\infty} \frac{2x^3+x}{3x^3-x^2+4}$

(c) $\displaystyle\lim_{x\to 0} x^2 \sin(1/x)$

[3] (10 pts)

Please leave blank!

[4] **(10 pts)**

(a) **(7 pts)** Using the limit definition of derivative, find $f'(x)$ for $f(x) = \sqrt{x}$.

(b) **(3 pts)** Find an equation for the tangent line to the graph $y = \sqrt{x}$ at the point $(1, 1)$.

[4] (10 pts)

Please leave blank!

[5] (10 pts)

(a) (5 pts) Let $F(x) = \int_0^{\tan(x)} \sqrt{1-t^3}\,dt$. Find $F'(x)$.

(b) (5 pts) Find $\frac{dy}{dx}$ for the curve $x^2 + 2xy - y^2 = 2$.

[5] (10 pts)

Please leave blank!

[6] **(10 pts)** For the function $f(x) = \dfrac{x^3}{x^2 - 1}$, you are given (do not compute!) that

$$f'(x) = \frac{x^2(x^2 - 3)}{(x^2 - 1)^2} \quad \text{and} \quad f''(x) = \frac{2x(x^2 + 3)}{(x^2 - 1)^3}.$$

(a) Find the domain of $f(x)$.

(b) Find the coordinates of all intercepts, and the equations of all horizontal and vertical asymptotes, of the graph of $y = f(x)$.

(c) In what intervals is the function f increasing? decreasing?

(d) In what intervals is the graph of f concave up? concave down?

(e) Find the coordinates of all local maxima, local minima, and points of inflection of the function f.

(f) Sketch the graph of $y = f(x)$. Label the features you found in items b and e.

[6] (10 pts)

Please leave blank!

[7] (5 pts each)

(a) Air is being pumped into a spherical balloon at a rate of $5\text{ft}^3/\text{min}$. Determine the rate at which the surface area of the balloon is increasing when the radius of the balloon is 20ft.

Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$, and the surface area of a sphere of radius r is given by $S = 4\pi r^2$.

(b) Use calculus to find the absolute maximum and minimum values of the function $f(x) = \sin(x) + \cos(x)$ for x in the interval $[0, \pi/2]$.

[7] (10 pts)

Please leave blank!

Part II, questions 8 to 12. Answer three out of those five questions, and cross out the two questions you would like to omit.

[8] **(5 pts each)** A particle moves along a straight line. At time $t = 0$ seconds, it is observed at position $s = -3$ meters moving at a speed of 5 meters per second in the positive direction.

(a) Use the given information to write down a linearization $L(t)$ of the position function $s(t)$.

(b) Use your answer to **(a)** to estimate the position of the particle at time $t = 2$ seconds.

[8] (10 pts)

Please leave blank!

[9] **(10 pts)**

(a) **(2 pts each)** Let $f(x) = \begin{cases} x & \text{if } x < 1, \\ 2 & \text{if } x = 1, \\ x^2 & \text{if } x > 1. \end{cases}$

(i) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 4$.

(ii) Find $\lim_{x \to 1} f(x)$.

(iii) Is $f(x)$ continuous at the point $x = 1$? Please justify your answer.

Question 9 continues on the next page.

(b) (4 pts) Using Riemann Sums, write down an expression that estimates $\int_0^4 f(x)dx$ using the Left Endpoint Rule with 4 subdivisions. You do not need to simplify this expression.

[**9**] (10 pts)

Please leave blank!

[10] (10 pts) We need to enclose a rectangular field with a fence, and we have 500 feet of fencing. One side of the field will be alongside a building and therefore will not require any fencing.

Determine the dimensions of the field that will enclose the largest area.

Note: Please justify your answer using calculus.

[10] (10 pts)

Please leave blank!

[11] **(10 pts)** Sketch the graph of an example of a continuous function $f : (0, +\infty) \to \mathbb{R}$ (the domain of this function is $(0, +\infty)$) that has **all** of the following properties:

(a) $f(1) = f(4) = 0$

(b) $\lim_{x \to 0^+} f(x) = +\infty$

(c) $\lim_{x \to +\infty} f(x) = 0$

(d) f has a local minimum at $x = 3$ and a local maximum at $x = 5$.

(e) f has two inflection points: one at $x = 3$, and one at $x = 6$.

[11] (10 pts)

Please leave blank!

[12] (10 pts)

(a) (6 pts) Compute each of the following limits or show that the limit does not exist.

(i) $\displaystyle\lim_{x\to\infty} \frac{\sin x \cos x}{x}$

(ii) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x+5}-3}{x-4}$

(iii) $\displaystyle\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$

(b) (4 pts) Show that the equation $x^7 + x - 1 = 0$ has one solution c that lies on the interval $(0, 1)$.

[12] (10 pts)

Please leave blank!