Exponential Growth and Decay Handout

Any quantity whose rate of change is proportional to its current size is said to have exponential growth (if its growing) or exponential decay (if it is decaying).

The formulas governing Exponential Growth

Equation 1 (The differential equation): $P' = rP$

The above has the initial condition $P(0) = P_0$.

Equation 2: $P = P_0 e^{rt}$

$$t_D = \frac{\ln 2}{r}$$

The formulas governing Exponential Decay

Equation 1 (The differential equation): $P' = -rP$

The above has the initial condition $P(0) = P_0$.

Equation 2: $P = P_0 e^{-rt}$

$$t_h = \frac{\ln 2}{r}$$

In the above formulas,
P - the current amount at time t,
P' - the rate of change of P,
r - the the growth constant (resp.) decay constant, sometimes called "(relative) growth rate"
P_0 - the initial amount,
t_D - the time to double (time it takes to double what you started with)
t_h - the half-life (time it takes to have half of what you started with)

Compound Interest

We will only deal with quantities or money being compounded "continuously". This is when your money grows exponentially, and hence, follows the first set of equations (those for Exponential Growth). With that interpretation, P is the amount of money in an account after time t, P_0 is the initial principal, and r is the interest rate.

EXPONENTIAL GROWTH, EXPONENTIAL DECAY AND COMPOUND INTEREST PROBLEMS.

From Spring 2009:

4) Let $P(t)$ be the population of a colony of bacteria. At 11AM there are 60 bacteria and at 3PM there are 350. Assume exponential growth. (9 points)

[3 pts] **a)** Find $P(t)$ and simplify.

[3 pts] **b)** What is the size of the population at 4PM?.

[3 pts] **c)** When will the population reach 2000?

8) A dosage of 3 mg of radioactive iodine I^{131} is administered for some forms of thyroid cancer. The half-life of I^{131} is 8 days. If the maximum permissible level of I^{131} in the thyroid is 5 mg, what is the minimum number of days before a second dosage of 3 mg can be given? (Assume I^{131} is totally absorbed by the thyroid and is not excreted by the body.)

From Fall 2009:

4) Let $P(t)$ be the population of a colony of bacteria. At 10AM there are 50 bacteria and at 3PM there are 350. Assume exponential growth. (9 points)

[3 pts] **a)** Find $P(t)$ and simplify.

[3 pts] **b)** What is the size of the population at 4PM?.

[3 pts] **c)** When will the population reach 1500?

11) A bank pays 5% interest compounded continuously. Suppose you make an initial deposit of $4000 in the account.

a) Write a differential equation together with initial condition whose solution gives the amount in your account at any future time.

b) At what rate (in $/year) is your account increasing when the principal reaches $6000.

c) How long will it take until the principal in the account reaches $5000?

From Spring 2010:

4) Let $P(t)$ be the population of a colony of bacteria. At 1PM there are 50 bacteria and at 3PM there are 350. Assume exponential growth. (14 points)

[4 pts] **a)** Find the differential equation satisfied by $P(t)$.

[2 pts] **b)** Find $P(t)$ and simplify.

[4 pts] **c)** What is the size of the population at 6PM?.

[4 pts] **d)** When will the population reach 1600?

EXPONENTIAL GROWTH, EXPONENTIAL DECAY AND COMPOUND INTEREST PROBLEMS, cont'd.

From Fall 2010:

4) The half-lift of censium-137 is 30 years. Suppose we have a 200 mg sample. Let $P(t)$ be the mass remaining after t years. (14 points)

[4 pts] **a)** Find the differential equation satisfied by $P(t)$.

[2 pts] **b)** Find $P(t)$ and simplify.

[4 pts] **c)** How much mass remains after 75 years?.

[4 pts] **d)** After how many years is the mass reduced to 1 mg?

More examples for practice:

1. Let $P(t)$ be the current size of the population of a colony of bacteria at time t. At 10am there are 50 bacteria and at 3pm there are 350. Assume the rate of growth of the population is proportional to its current size.
   (a) Find the relative growth rate of the population.
   (b) Find a formula for $P(t)$.
   (c) What is the size of the population at 4pm?
   (d) What is the rate of growth at 4pm?
   (e) When will the population reach 1500?

2. The half-life of a radioactive substance is 30 years. Suppose we have a 200 mg sample of the substance. Let $P(t)$ be the mass remaining after t years.
   (a) Find the differential equation satisfied by $P(t)$.
   (b) Find a formula for $P(t)$.
   (c) How much will remain after 75 years?
   (d) After how many years will the mass be reduced to 1 mg?
   (e) What is the rate of growth at the time of (d)?