

The City College Department of Mathematics

Fall 2013 Math 20200 Final Exam

This is a closed-book and closed-note examination.

CALCULATORS are NOT allowed.

Please show all your work.

Use only the paper provided. You may write on the back if you need more space, but clearly indicate this on the front.

PART I: Answer ALL questions in this part. (70 points)

PART II: Answer three complete questions out of five. Each question is worth 10 points. If you answer more than three questions, cross out work you do not want graded.

Part 1 (questions 1 to 5): Answer all questions (70 points)

1. Find the derivative for each of the following and simplify (6 points each).

$$(a) y = \frac{5^x \sqrt[4]{x^3}}{x^7(x+1)^2} \quad (b) y = \sinh(\arcsin \sqrt{x}) \quad (c) y = x(\ln x)^x$$

2. Compute each of the following integrals (6 points each).

$$(a) \int \frac{x+4}{x^2+2x+5} dx \quad (c) \int \frac{x^2}{\sqrt{1-x^6}} dx$$
$$(b) \int_{\pi/6}^{\pi/4} \cos^2(x) \sin(2x) dx \quad (d) \int_1^{\sqrt{3}} \arctan \frac{1}{x} dx$$

3. Evaluate the limits (4 points each).

(a) $\lim_{x \rightarrow 0^+} (\tan x)^{2x}$

(b) $\lim_{x \rightarrow -\infty} e^x \sqrt[3]{x}$

4. (12 points) The region R is bounded by the curves $y = x^2 + 1$, and $y = 3x - 1$. Set up two integrals (method of washers and method of shells) for the volume of the solid obtained by rotating R around the line $x = 4$. Use one of these to compute the volume.

5. (8 points) The is curve given parametrically by $x = e^t + e^{-t}$, $y = 5 - 2t$.

(a) For which values of t the curve is concave upward?

(b) Find the exact length of the curve for $0 \leq t \leq 3$

END OF PART I

Part II: Answer 3 complete questions (10 points each)

6. A 1600-pound elevator is suspended by a 200-foot cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

7.

(a) Sketch the curve given by the equation $r = 1 - 3 \sin(\theta)$ in polar coordinates. Find the slope of the tangent line to the curve at the point where $\theta = \frac{\pi}{3}$.

(b) A freshly brewed cup of coffee has temperature 95°C in a 20°C room. If the temperature of coffee is 70°C after 20 minutes, what is its temperature after another 20 minutes? (Leave your answer as a reduced fraction.)

8.

(a) Evaluate the integral or show that it is divergent: $\int_0^3 \frac{1}{x^2 + x - 6} dx$.

(b) Use the Midpoint rule for 4 intervals to approximate

$$\int_{\frac{\pi}{12}}^{\frac{3\pi}{4}} \sqrt{1 + \cos x} dx.$$

(Evaluate the trigonometric functions and leave your answer in terms of square roots)

9.

(a) Draw a sketch of the conic whose equation is

$$4x^2 - 16x + y^2 + 6y + 21 = 0.$$

Identify which sort of conic it is. On your sketch, show and label whichever of the following are present: vertices, asymptotes, and foci.

(b) Sketch the curves $y = \ln x$, $y = 2 - 2x$ and $x = 3$. Compute the area of the entire bounded region which has these three curves as boundaries.

10. Evaluate

$$\int \frac{x^6 + x^4 + x^2 - 1}{x(x^2 + 1)^2} dx$$