Math 20300
Calculus III

Lesson 10
Parametric Surfaces

Dr. A. Marchese, The City College of New York

Bookmarks have been added to this video at the following times:

pagebreak pagebreak pagebreak pagebreak pagebreak pagebreak pagebreak pagebreak pagebreak

Parametric Surfaces

In Lesson 9 we graphed curves in $\mathbb{R}^3$ by mapping $\mathbb{R}$ into $\mathbb{R}^3$.

$$t \longmapsto (x(t), y(t), z(t))$$

In this lesson, we map $\mathbb{R}^2$ into $\mathbb{R}^3$, and get surfaces in $\mathbb{R}^3$.

$$(u,v) \longmapsto (x(u,v), y(u,v), z(u,v))$$

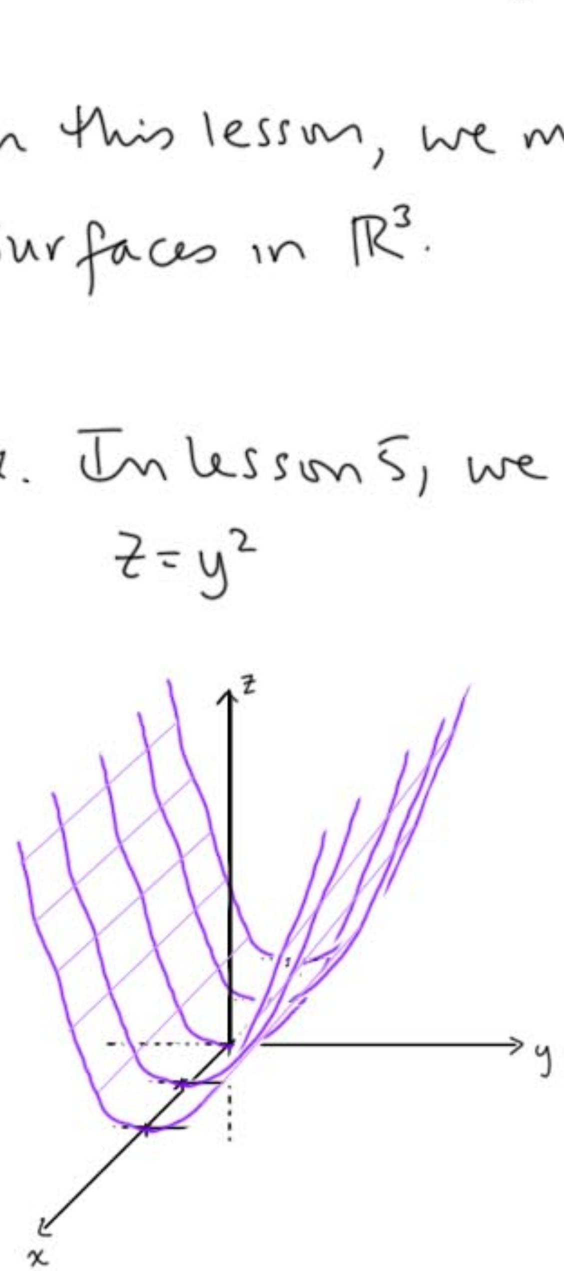
Ex. In lesson 5, we saw the Parabolic Cylinder

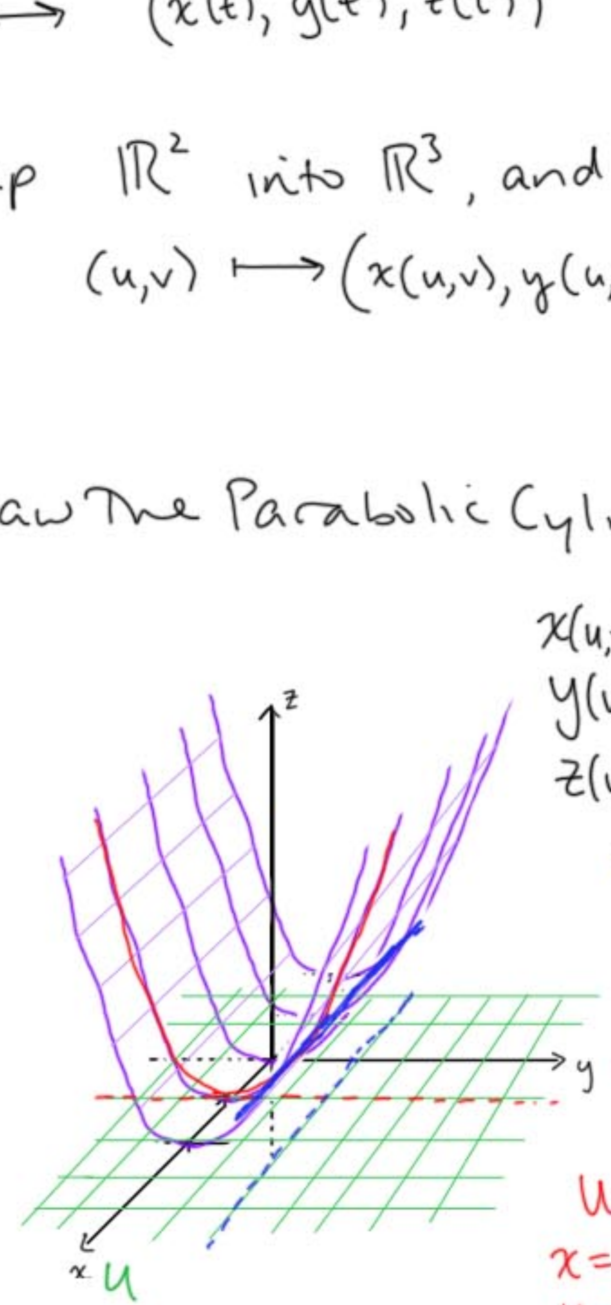
$z = y^2$

$x(u,v) = u$

$y(u,v) = v$

$z(u,v) = v^2$





$v = 2$

$x = u$

$y = 2$

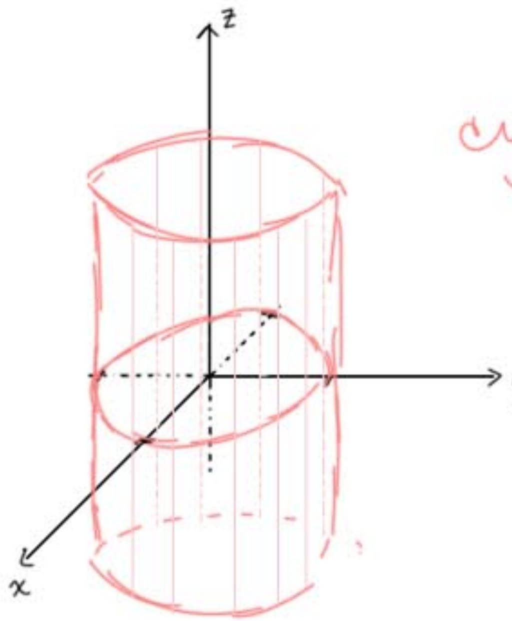
$z = 4$

$u = 1$

$x = 1$

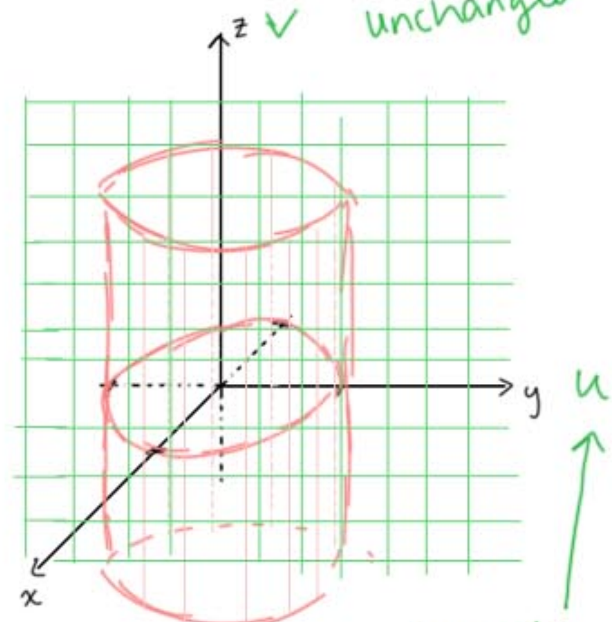
$y = v$, $z = v^2$

Ex. also from lesson 5



cylinder

$x^2+y^2=1$



how do we get a circle from one parameter? $\sin u, \cos u$

eliminate parameters

$x^2+y^2=\sin^2 u+\cos^2 u$
$=1$

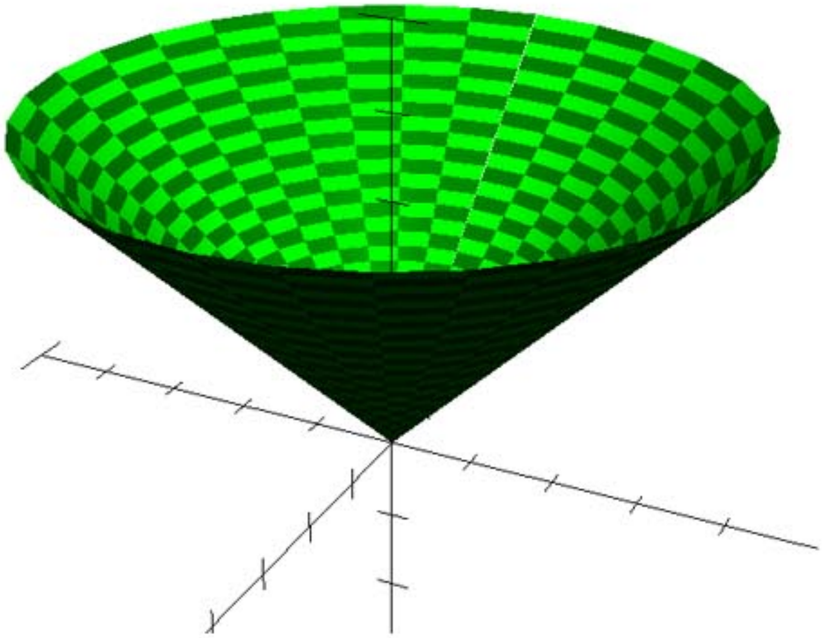
$x^2+y^2=1$

let $x(u,v)=\sin u$
$y(u,v)=\cos u$
$z(u,v)=v$

Ex. Identify the surface by eliminating the parameters:

$x(u,v)=u\cos v \qquad 0\le u\le 2$
$y(u,v)=u\sin v \qquad 0\le v\le 2\pi$
$z(u,v)=u$

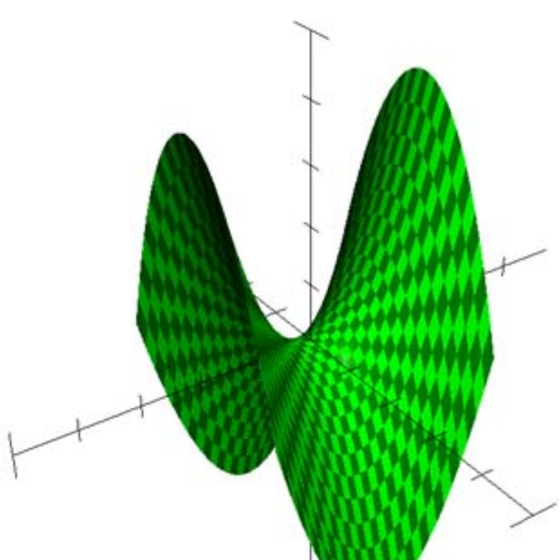
$x^2+y^2=u^2(\cos^2 v+\sin^2 v)=u^2$
$x^2+y^2=z^2$ cone with $0\le z\le 2$.



Ex. Identify the surface by eliminating the parameters

$$x(u,v) = u$$
$$y(u,v) = v$$
$$z(u,v) = u^2 - v^2$$
$$z = x^2 - y^2$$

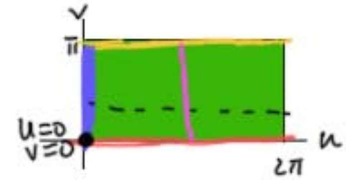
hyperbolic paraboloid



Ex. $x(u,v) = 2\cos u \cos v \qquad 0 \le u \le 2\pi$

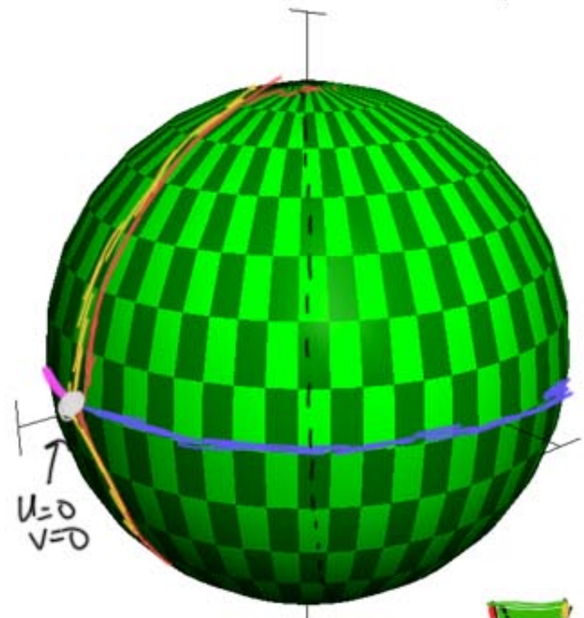
$y(u,v) = 2\cos u \sin v \qquad 0 \le v \le \pi$

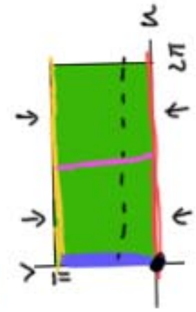
$z(u,v) = 2\sin u$

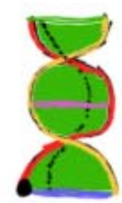


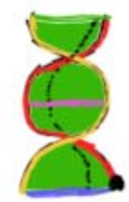
When $v=0$	$v=\pi$
$x = 2\cos u$	$x = -2\cos u$
$y = 0$	$y = 0$
$z = 2\sin u$	$z = 2\sin u$

circle









When $u=0$

$x = 2\cos v$

$y = 2\sin v$ semicircle for $0 \le v \le \pi$

(front)

$z = 0$

When $u = \pi$

$x = -2\cos v$

$y = -2\sin v$ semicircle for $0 \le v \le \pi$

(back)

$z = 0$

Eliminating the parameters:

$$x(u,v) = 2\cos u \cos v$$
$$y(u,v) = 2\cos u \sin v$$
$$z(u,v) = 2\sin u$$

$$x^2+y^2 = (2\cos u \cos v)^2 + (2\cos u \sin v)^2$$

$$= 4\cos^2 u \cos^2 v + 4\cos^2 u \sin^2 v$$

$$= 4\cos^2 u \, (\underbrace{\cos^2 v + \sin^2 v}_{1})$$

$$= 4\cos^2 u$$

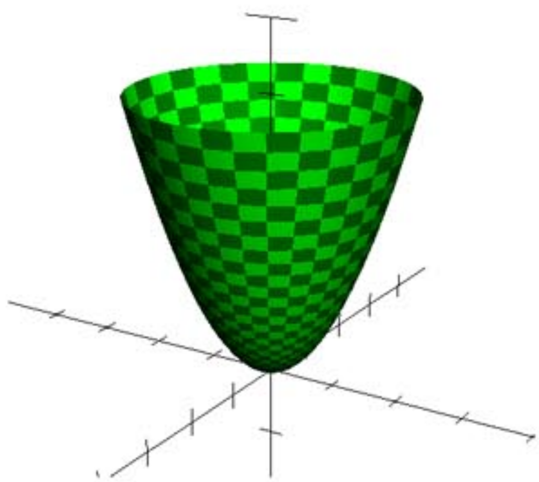
$$x^2+y^2+z^2 = 4\cos^2 u + 4\sin^2 u = 4$$

$x^2+y^2+z^2 = 4$ sphere of radius 2 at origin.

Ex. Find a parametric representation for the surface:

The portion of $z = \underbrace{x^2+y^2}_{\text{circular}}$ below $z=4$

paraboloid:



$$x = v\sin u \qquad 0 \le u \le 2\pi$$

$$y = v\cos u \qquad 0 \le v \le 2$$

$$z = v^2$$

$$\begin{aligned} x^2+y^2 &= v^2\sin^2 u + v^2\cos^2 u \\ &= v^2(1) = v^2 \\ &= z \end{aligned}$$

Ex. Find a parametric representation for

the portion of the ellipsoid

$$9x^2 + 36y^2 + 4z^2 = 36$$

that is in front of the xz plane.

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1$$

Compare to sphere above

sphere of radius 2:

$$x(u,v) = 2\cos u \cos v$$
$$y(u,v) = 2\cos u \sin v$$
$$z(u,v) = 2\sin u$$
$$0 \le u \le 2\pi$$
$$0 \le v \le \pi$$

our ellipse:

$$x(u,v) = 2\cos u \cos v$$
$$y(u,v) = \cos u \sin v$$
$$z(u,v) = 3\sin u$$
$$-\pi/2 \le u \le \pi/2$$
$$0 \le v \le \pi$$