# Math 20300
# Calculus III

# Lesson 2
# Vectors

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# Vectors

A <u>vector</u> quantity is one that has both magnitude and direction

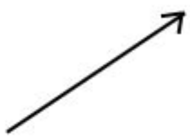
Compare to a <u>scalar</u> quantity, which only has magnitude.

Ex. 10 miles scalar

10 miles North vector

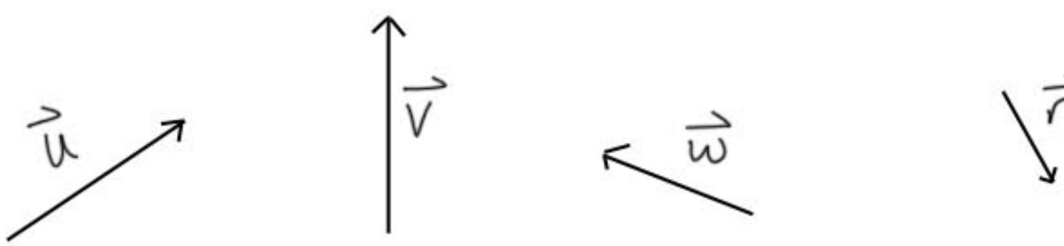
Other examples of vector quantities: velocity, acceleration, force.

graphic representation of a vector quantity is a directed line segment:



Length of the line segment represents the magnitude of the vector, and the placement and arrow show the direction

Examples of vectors:

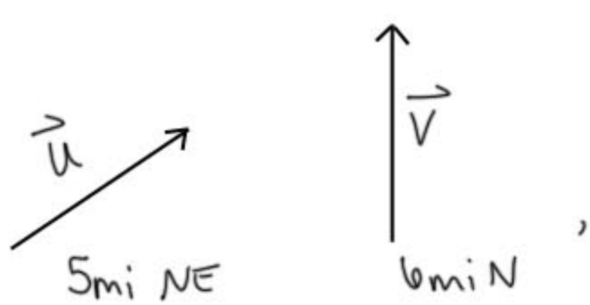



vector notation: $\vec{u}$ "vector hat"

or $\mathbf{u}$ bold type

Addition of vectors:

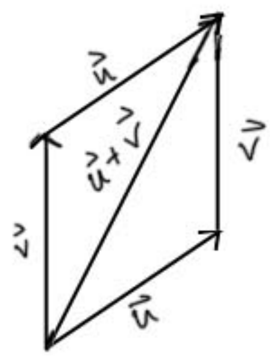
given



$\vec{u} + \vec{v}$ :

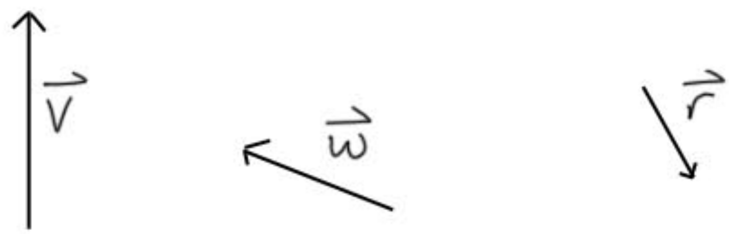
placement is "tail to tip" when adding vectors

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$




Ex.

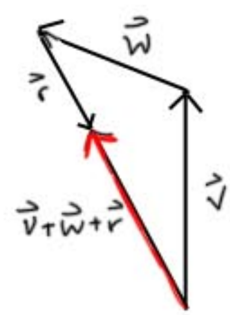
given




,

find $\vec{v} + \vec{w} + \vec{r}$

tail to tip




Scalar multiplication of a vector:

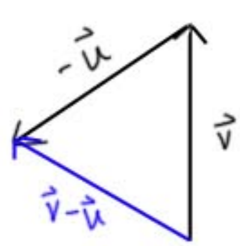
given $\vec{u}$, what does $2\vec{u}$ represent?

the scalar multiplication of 2 gives a resulting vector with twice the magnitude, in the same direction


vectors are said to be <u>parallel</u> if they are scalar multiples of each other.

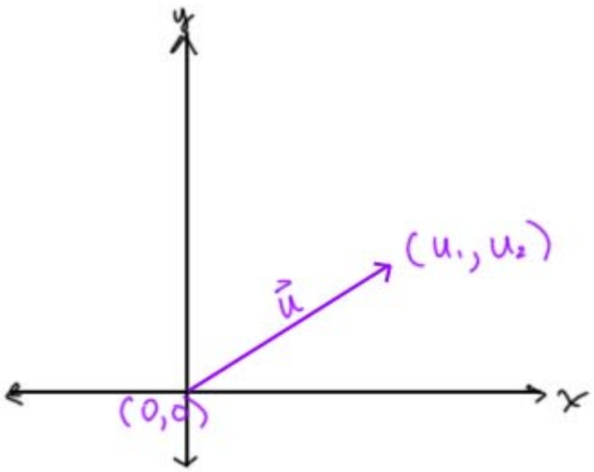

the negative sign changes the direction

Subtraction of vectors: $\vec{v} - \vec{u} = \vec{v} + (-\vec{u})$



Vectors in $\mathbb{R}^2$ + $\mathbb{R}^3$

plotting and notation




we can plot vectors with any starting or ending point that is convenient for us.

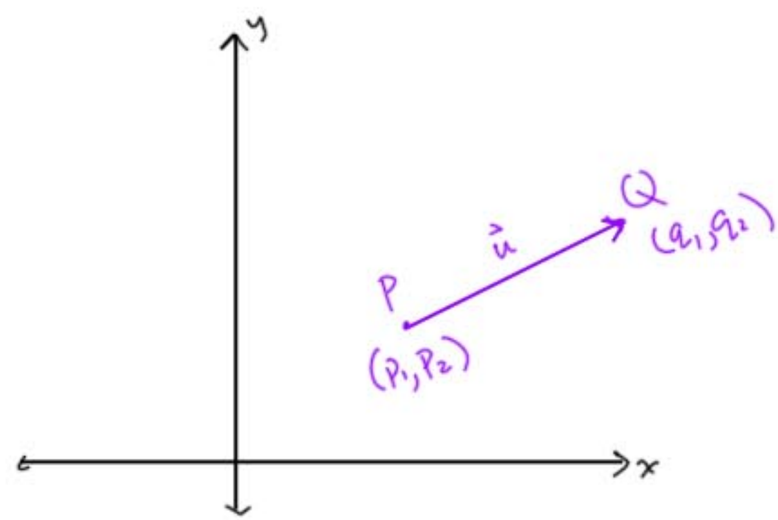
initial point for vector $\vec{u}$ is $(0,0)$

terminal point for vector $\vec{u}$ is $(u_1, u_2)$

Component notation: $\vec{u} = \langle u_1, u_2 \rangle$

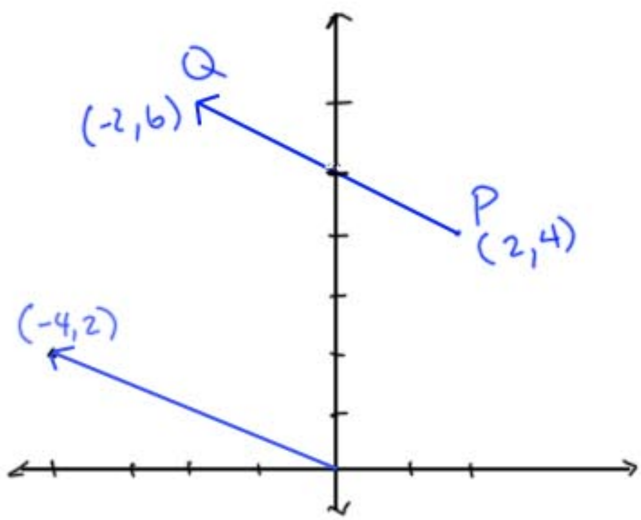
where $(u_1, u_2)$ is the terminal point of vector $\vec{u}$ when initial point is $(0,0)$

given vector $\vec{u}$ elsewhere in the plane, how do we get component notation?




$$\vec{u} = \overrightarrow{PQ} = \langle q_1 - p_1 \, , q_2 - p_2 \rangle$$

Ex. find the component notation for vector PQ:




$$\overrightarrow{PQ} = \langle q_1 - p_1 , \; q_2 - p_2 \rangle$$

$$= \langle -2 - 2 , \; 6 - 4 \rangle$$

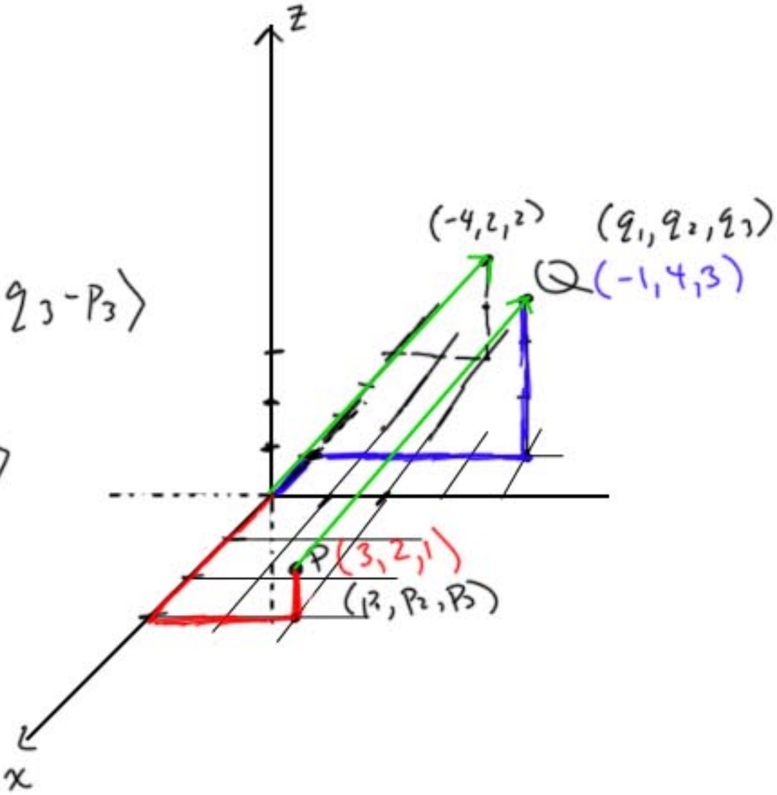
$$= \langle -4 , 2 \rangle$$

Vectors in $\mathbb{R}^3$

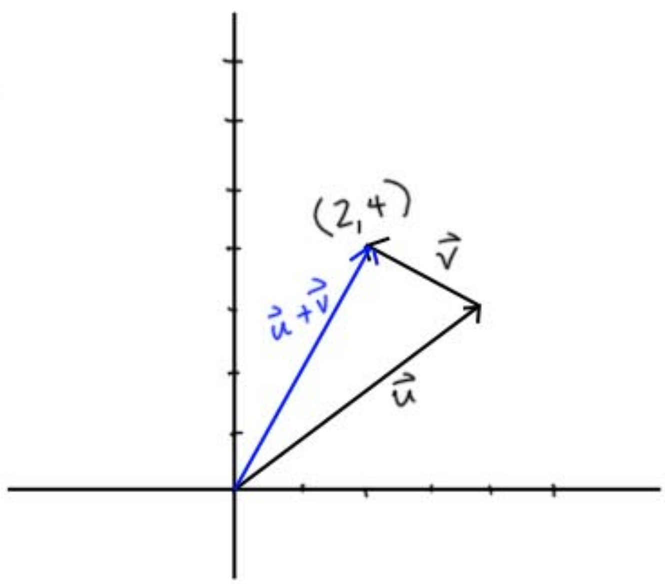
$\overrightarrow{PQ} = \langle q_1 - p_1,\ q_2 - p_2,\ q_3 - p_3 \rangle$

$= \langle -1-3,\ 4-2,\ 3-1 \rangle$

$= \langle -4, 2, 2 \rangle$




Ex.




$\vec{u} = \langle 4, 3 \rangle$

$\vec{v} = \langle -2, 1 \rangle$

$\vec{u} + \vec{v} = \langle 2, 4 \rangle$

$= \langle 4-2,\ 3+1 \rangle$

adding vectors component-wise gives us the components of the resultant vector.

Given vectors $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$

then $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$

Given vectors $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$

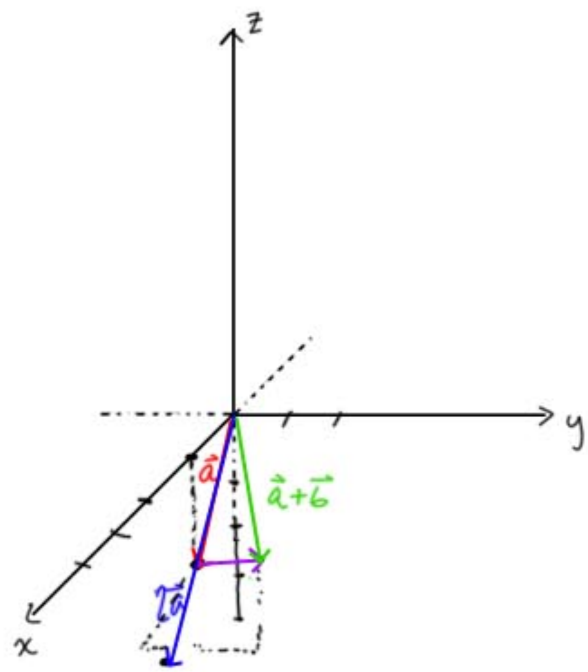
Scalar multiplication $\vec{u} = \langle u_1, u_2 \rangle$ or $\vec{u} = \langle u_1, u_2, u_3 \rangle$

$c\vec{u} = \langle cu_1, cu_2 \rangle$ or $c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$

Compute + sketch

$2\vec{a} = \langle 2, 0, -4 \rangle$

$\vec{a} + \vec{b} = \langle 3, 2, 0 \rangle$



Ex. $\vec{a} = \langle 1, 0, -2 \rangle$ $\vec{b} = \langle 2, 2, 2 \rangle$

1) $\vec{a} + \vec{b} = \langle 1+2, 0+2, -2+2 \rangle = \langle 3, 2, 0 \rangle$

2) $2\vec{a} - \vec{b} = \langle 2(1)-2, 2(0)-2, 2(-2)-2 \rangle$

$= \langle 0, -2, -6 \rangle$

3) $\vec{a} + 3\vec{b} = \langle 1 + 3(2), 0 + 3(2), -2 + 3(2) \rangle$

$= \langle 7, 6, 4 \rangle$

Length of vectors

$\vec{v} = \langle v_1, v_2 \rangle$ or $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$ or $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

by the distance formula.

notation: $\|\vec{v}\|$ or $|\vec{v}|$ represents the length of $\vec{v}$, same as magnitude of $\vec{v}$.

Ex. Find the length of the vector:

$\vec{a} = \langle 2, 4 \rangle \qquad \vec{b} = \langle 1, 3, -2 \rangle$

$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2} \quad$ or $\quad \|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

STOP Work on this problem on your own

$\|\vec{a}\| = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$\|\vec{b}\| = \sqrt{1^2 + 3^2 + (-2)^2} = \sqrt{1+9+4} = \sqrt{14}$

Ex. For $\vec{V} = \langle v_1, v_2, v_3 \rangle \quad \|\vec{V}\| = \sqrt{V_1^2 + V_2^2 + V_3^2}$

Find $\|c\vec{v}\|$

$c\vec{V} = \langle cv_1, cv_2, cv_3 \rangle$

$\|c\vec{v}\| = \sqrt{(cv_1)^2 + (cv_2)^2 + (cv_3)^2} = \sqrt{c^2(v_1^2 + v_2^2 + v_3^2)}$

$= |c| \sqrt{v_1^2 + v_2^2 + v_3^2}$

$= |c| \|\vec{v}\|$.

A vector with length $=1$ is called a <u>unit vector</u>.

Vector $\vec{b}$ above $\vec{b} = \langle 1, 3, -2 \rangle$ and $\|\vec{b}\| = \sqrt{14}$

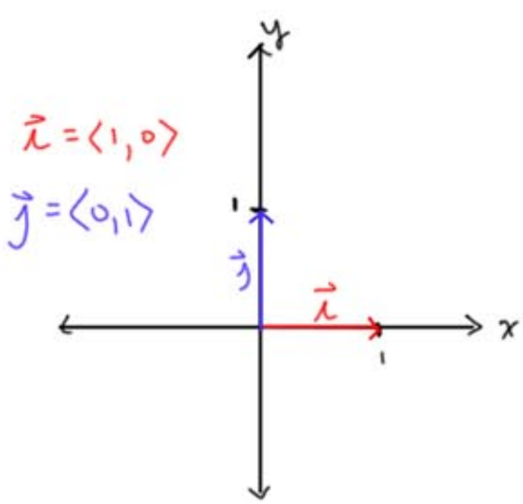
Find a unit vector in the direction of $\vec{b}$.

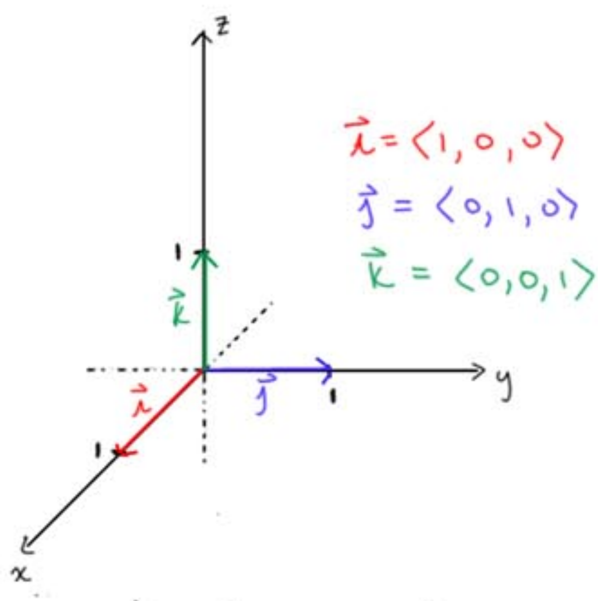
$$\vec{u} = \frac{1}{\sqrt{14}}\vec{b} = \frac{1}{\sqrt{14}}\langle 1, 3, -2 \rangle = \left\langle \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}} \right\rangle$$

$$= \left\langle \frac{\sqrt{14}}{14}, \frac{3\sqrt{14}}{14}, \frac{-2\sqrt{14}}{14} \right\rangle = \left\langle \frac{\sqrt{14}}{14}, \frac{3\sqrt{14}}{14}, \frac{-\sqrt{14}}{7} \right\rangle.$$

$$\boxed{\text{unit vector in direction of } \vec{v}: \quad \vec{u} = \frac{\vec{v}}{\|\vec{v}\|}}$$

Basis vectors $\vec{i}, \vec{j}, \vec{k}$




$$\vec{v} = \langle v_1, v_2 \rangle = v_1\vec{i} + v_2\vec{j}$$

$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$
$$= v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$$

Ex. $\vec{a} = \langle 2, -3 \rangle$ $\qquad\qquad$ $\vec{b} = \langle 4, 0, -3 \rangle$

$= 2\vec{i} - 3\vec{j}$ $\qquad\qquad$ $= 4\vec{i} + 0\vec{j} - 3\vec{k}$

$= 4\vec{i} - 3\vec{k}$

<u>Zero vector.</u> $\quad \vec{0} = \langle 0, 0 \rangle \qquad \vec{0} = \langle 0, 0, 0 \rangle$

Ex. $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k} \qquad \vec{b} = -\vec{i} + 3\vec{j} \qquad \vec{c} = -\vec{i} - \vec{k}$

$= \langle 2, -3, 1 \rangle \qquad\qquad = \langle -1, 3, 0 \rangle \qquad\qquad = \langle -1, 0, -1 \rangle$

1) $2\vec{a} + \vec{b} = \langle 2(2) - 1,\ 2(-3) + 3,\ 2(1) + 0 \rangle$

$= \langle 3, -3, 2 \rangle = 3\vec{i} - 3\vec{j} + 2\vec{k}$

2) $\vec{a} + \vec{b} + \vec{c} = \langle 2 - 1 - 1,\ -3 - 3 + 0,\ 1 + 0 - 1 \rangle = \langle 0, 0, 0 \rangle$

$= 0\vec{i} + 0\vec{j} + 0\vec{k} = \vec{0}$

3) $\|\vec{a} - \vec{c}\| \qquad \vec{a} - \vec{c} = \langle 2 - {}^-1,\ -3 - 0,\ 1 - {}^-1 \rangle$

$= \langle 3, -3, 2 \rangle$

$\|\vec{a} - \vec{c}\| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9 + 9 + 4}$

$= \sqrt{22}$.