Math 20300
Calculus III

Lesson 1
The Three Dimensional Coordinate System

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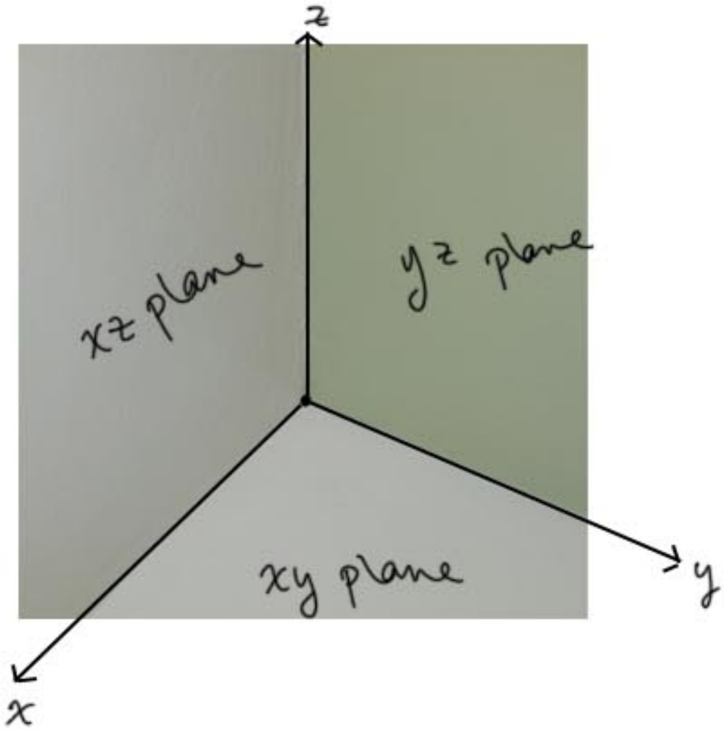
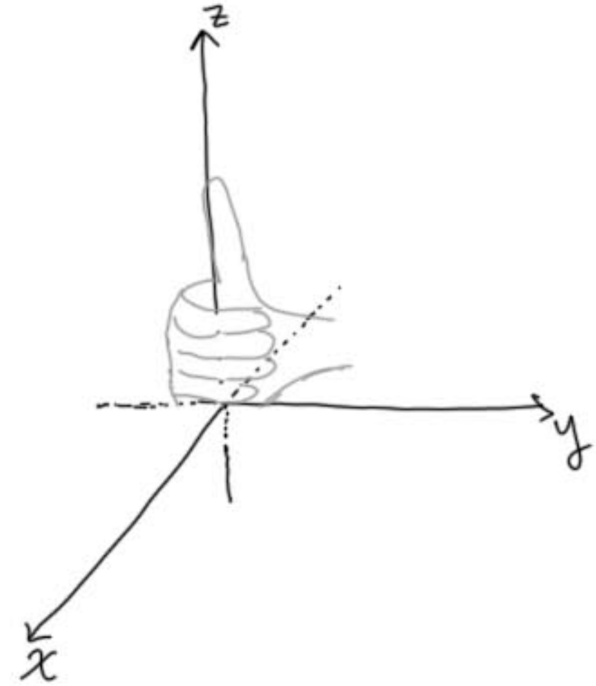
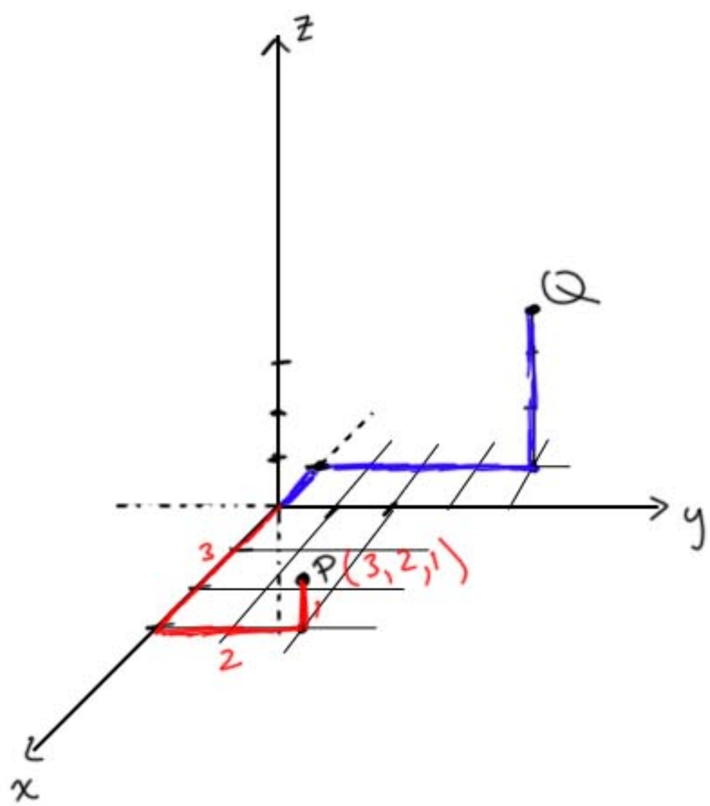
Three-Dimensional Coordinate System







plot (3, 2, 1) P
(x, y, z)

plot Q (−1, 4, 3)

$\mathbb{R}$ real numbers

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x, y) \mid x, y \in \mathbb{R}\}$$

= set of all ordered pairs of real numbers

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$$

= set of all ordered triples of real numbers.

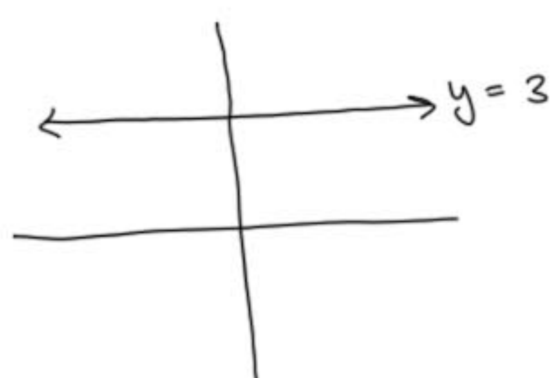
$\mathbb{R}^2$, axes divide the plane into quadrants

$\mathbb{R}^3$, axes define octants

in particular, the 1st octant is defined by the positive x, y and z axes

Ex. $\mathbb{R}^2$ $\quad y = 3$

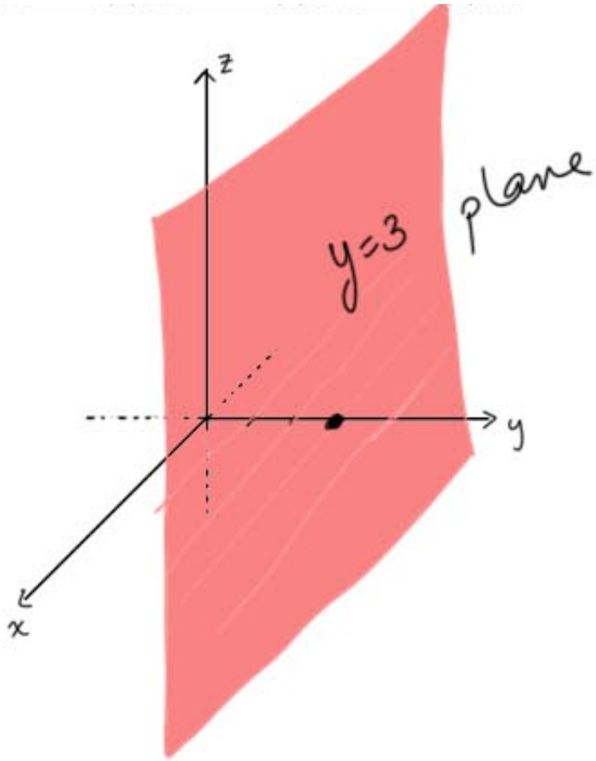
all points in $\mathbb{R}^2$ with y coordinate 3



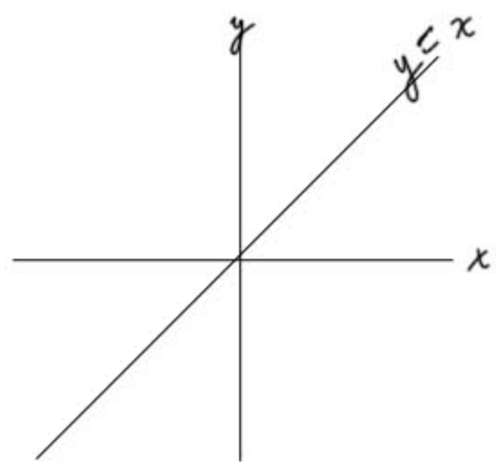
$\mathbb{R}^3$ $\quad y = 3$

all points in $\mathbb{R}^3$

with y cordinate 3

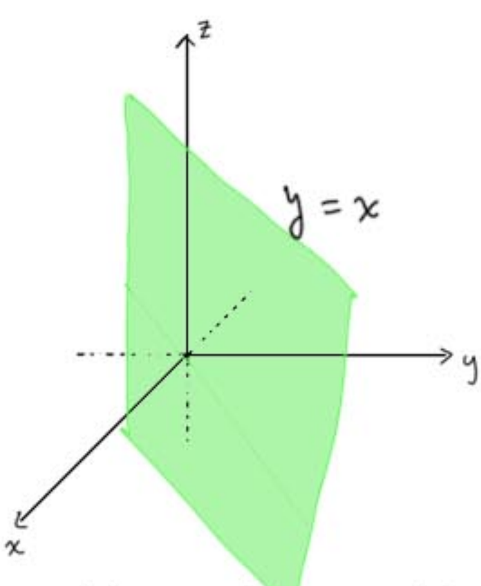


$\mathbb{R}^2$ $\quad y = x$ line

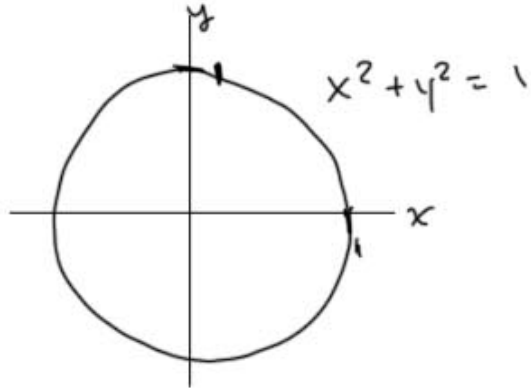


$\mathbb{R}^3$

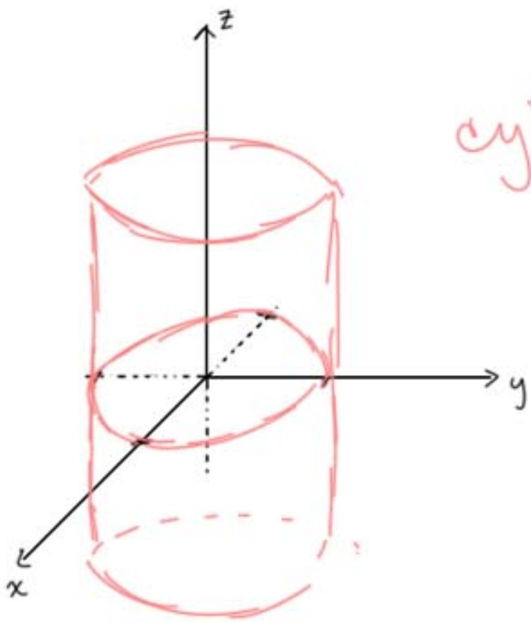
$y = x$



$\mathbb{R}^2$ $\quad x^2+y^2=1$



$\mathbb{R}^3$

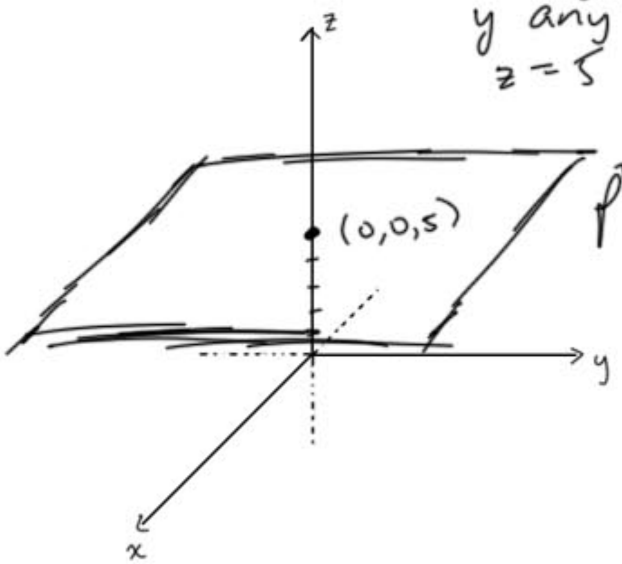


cylinder

$x^2+y^2=1$ in $\mathbb{R}^3$

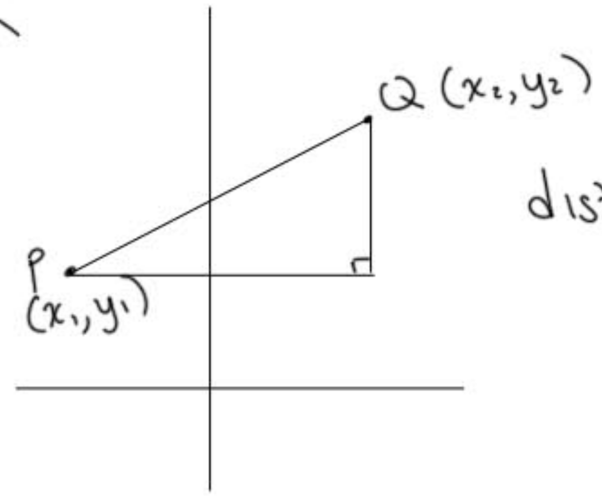
$\mathbb{R}^3$ $\quad z=5$

x any value
y any value
z = 5



plane
$z=5$

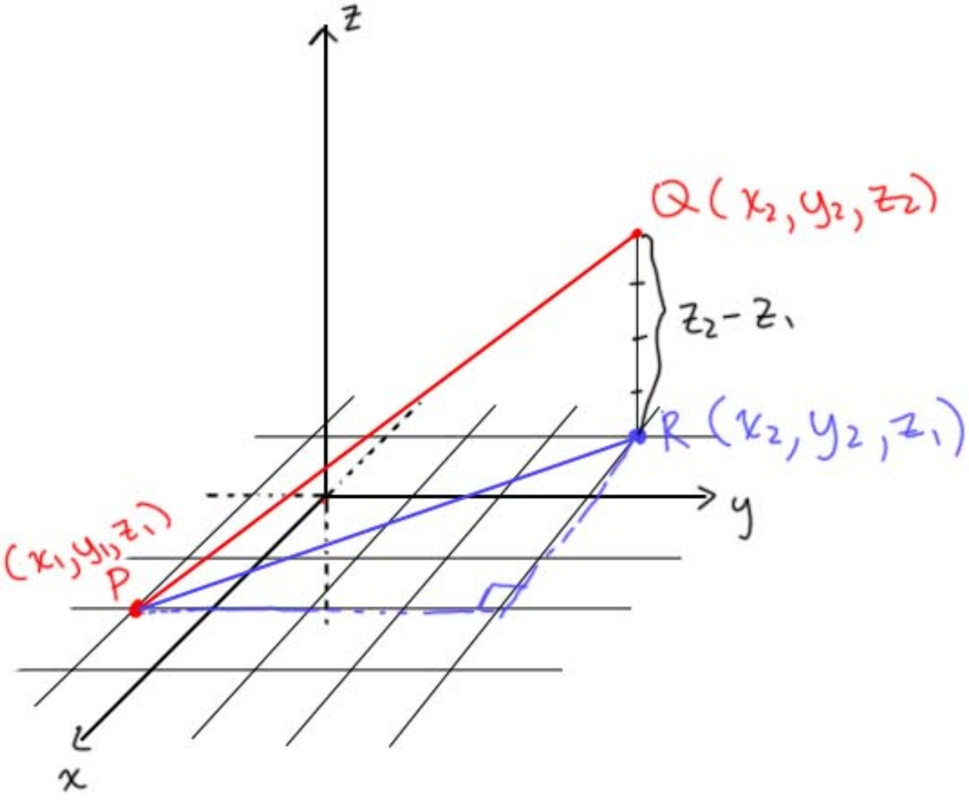
$\mathbb{R}^2$



$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

from pythagorean theorem

$\mathbb{R}^3$



$$|PR| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(|PR|)^2 + (z_2-z_1)^2}$$

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}.$$

$\therefore$ Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Ex.

if P $(2,-1,0)$ and Q $(-1,3,4)$

$$|PQ| = \sqrt{(-1-2)^2 + (3--1)^2 + (4-0)^2} = \sqrt{9+16+16}$$

$$= \sqrt{41}.$$

Ex. $P(3,-1,2)$ Is $\triangle PQR$ equilateral?

$Q(1,1,2)$

$R(1,-1,4)$

STOP Work on this problem on your own

$$|PQ| = \sqrt{(1-3)^2+(1-{-1})^2+(2-2)^2} = \sqrt{4+4+0} = \sqrt{8}$$

$$|QR| = \sqrt{(1-1)^2+(-1-1)^2+(4-2)^2} = \sqrt{0+4+4} = \sqrt{8}$$

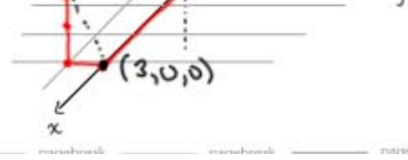
$$|PR| = \sqrt{(1-3)^2+(-1-{-1})^2+(4-2)^2} = \sqrt{4+0+4} = \sqrt{8}$$

Yes, $\triangle PQR$ is equilateral

Ex. Find the distance from point $(3,-1,2)$

to a) the yz plane

b) the x axis



a) yz plane $\Leftrightarrow$ $x=0$

point is 3 units from yz plane

b) $\sqrt{(3-3)^2+(-1-0)^2+(2-0)^2} = \sqrt{0+1+4} = \sqrt{5}.$

Spheres

in $\mathbb{R}^2$, circle = set points equidistant from a given center point (h,k)

r, radius

in $\mathbb{R}^3$, sphere = set of points equidistant from a given center point (h,k,l)

r, radius

for (x,y,z) on sphere,

$$\sqrt{(x-h)^2+(y-k)^2+(z-l)^2} = r$$

$$(x-h)^2+(y-k)^2+(z-l)^2 = r^2$$

Equation of the sphere centered at (h,k,ℓ)

with radius r : $(x-h)^2+(y-k)^2+(z-\ell)^2 = r^2$

Ex. Find the center and radius of

$$x^2-4x+y^2+8y+z^2-6z=3$$

$$x^2\underbrace{-4}_{\div 2}x+4+y^2\underbrace{+8}_{\div 2}y+16+z^2\underbrace{-6}_{\div 2}z+9=3+4+16+9$$

$$(x-2)^2 \quad + \quad (y+4)^2 \quad + \quad (z-3)^2 \quad = \quad 32$$

$$x^2-4x+4 \qquad y^2+8y+16 \qquad z^2-6z+9$$

$$(x-2)^2+\underset{(y--4)^2}{(y+4)^2}+(z-3)^2=32$$

Center $(2,-4,3)$ radius $=\sqrt{32}$.