Here's a solution to problem 4(b) that does not use induction. I thought of this after writing up the solutions. I couldn't bother editing the solutions, but to my pleasant surprise, another student came up with a similar solution, so I decided to share it with you.

Theorem: (Denseness of $\mathbb{Q}$) If $a, b \in \mathbb{R}$ and $a < b$, then there exists $r \in \mathbb{Q}$ such that $a < r < b$.

(10 points) Prove that for any $a, b \in \mathbb{R}$, with $a < b$, there are an infinite number of rational numbers strictly between a and b. You may or may not use the denseness of $\mathbb{Q}$ theorem stated above. You also may or may not use induction here.

Pf: Assume to the contrary that there are not an infinite number of rationals strictly between a and b. We know there is at least one, by the denseness of $\mathbb{Q}$ theorem, so let us suppose there is only a finite amount. Say there are n (and no more) rationals between a and b. That is, we have rationals $r_1, r_2, \ldots, r_n$ such that $a < r_n < r_{n-1} < \ldots < r_2 < r_1 < b$, and there are no more such rationals. Then, applying the denseness of $\mathbb{Q}$ theorem to a and r_n, there exists a rational r_{n+1} such that $a < r_{n+1} < r_n$. But that gives $a < r_{n+1} < r_n < \ldots < r_1 < b$. So we found another rational between a and b not in our original list. Contradiction.

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