The Spiteful Computer: A Determinism Paradox

Ethan Akin

I was an undergraduate at CCNY when I first encountered Russell's paradox as an exercise in Simmons's topology book. I distinctly remember my enjoyment of the looping symmetry of the argument. Delightful. However, a full week passed, and then I suddenly realized that the paradox had not been resolved. Considerably agitated, I hunted up my professor, Jesse Douglas, who told me some home truths about logic and sent me off to read Halmos's "Naive Set Theory."

That was many years ago. Lately, I have been troubled by another paradox. Again I can't resolve it, but Professor Douglas is long dead. So I am presenting this description, not as a challenge, but as an appeal for help.

It began with some thoughts about classical determinism *a la* Laplace out of Newton. Determinism seems to exclude the possibility of free will; and indeed some behaviorists, like B. *F.* Skinner, suggest that our sense of free will is an illusion arising from incomplete information, just as the subject of a post-hypnotic suggestion feels completely uncoerced as he opens his umbrella in the living room, but we who saw the suggestion planted know better. I think the Skinner view is erroneous as well as demoralizing, but like other skeptical positions it is hard to attack by logical argument. Thus, I was delighted when someone pointed out to me that the discussion of spite by Dostoevsky's Underground Man provides an answer to the behaviorist view. When presented with the choice between coffee and tea, I can always, regardless of my tastes, simply out of spite, change my choice when someone attempts to publicly predict my behavior. I was very taken with this argument for the reality of human choice and I trotted it about, showing it off. Finally, I displayed it to my Math Department colleague, Morton Davis. He was not impressed. Shrugging, he remarked: "I can program a computer to do that." Thus crumbled my defense of free will. What remains is a puzzle about determinism.

Imagine a system consisting of two computers. The first, very large, is labeled "Laplace," and the second, rather small, is "Baby Dostoevsky." We assume a Newtonian world consisting, at the microscopic level, of atoms whose motion is completely described by a system of second-order differential equations. Computer Laplace is designed to solve the associated initial-value problem, but its programmed goal is to predict the output of Baby Dostoevsky at time $T + 1$. It prints this prediction at time T and inputs it to B.D. who has a very simple program: "No." So the output at time $T + 1$ is 1 if the input at T is 0 and is 0 otherwise.

We start the clock running at time $t = 0$, providing Laplace with the initial position and velocity of all of the particles in the system. By computing the associated solution path, it can observe the state of B.D. at all future times. Notice that the gadgets and their programming are included in the system about which the computations are being run. In particular, Laplace can observe the output of B.D. at T + 1. It prints this state at time $t = T$ and hands it on. But the programming of B.D. then falsifies the prediction.

There is a paradox here. This is a completely deterministic world. Pause here to think microscopically. What we interpret as two computers with programming embodied by switches and connecting wires, is, in fact, a vast array of particles moving about, attracting, and colliding with motions described by a system of equations we are assumed to know. The initial po-

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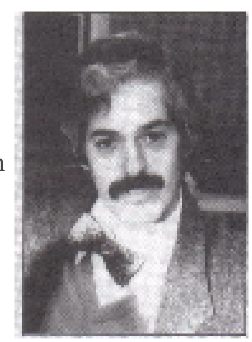

Graduating from CCNY in 1965, *Ethan Akin* returned there to teach after acquiring a PhD from Princeton and a bit of seasoning at UC, Berkeley. His interests are piecewise-linear topology, dynamical systems, philosophy of science, and horses. He describes this article as the fruit of twenty years' inexperience with actual computers.

A Spiteful Human

sitions and velocities determine the future states. So solving the initial value problem provides a prediction of the future. This was the original vision of Laplace (the human). In particular, all of the future outputs of B.D. were determined by the initial data and so are available to the solver, Laplace. "Having obtained this T + 1 output, print it and input it to B.D. at time T," is a macroscopic description of an aspect of the motion of the system, built-in to begin with and so also determined by the initial conditions. But clearly the logic of the programming makes this internal prediction impossible.

Clearly, there is no issue of free choice here. From the outside what is happening is so simple as to be dull. The whole issue is predictability from the inside. Furthermore, the possibility of a hypothetical prediction of the "Whatever I say, you will do the opposite" variety is simply irrelevant, as is the observation that Laplace can correctly predict the output by lying about its prediction to B.D. These objections miss the point and misunderstand the predetermined, hardwired nature of the world setup. Also, there seems to be a cliche that a system cannot predict its own state, but I don't know what this means. Take a piece of wire, bend it to spell out, in script, the word "Frozen," and then freeze it. If you want something less trivial, replace Baby Dostoevsky by Baby Dale Carnegie whose program is "Yes." Now the prediction works just fine.

I am aware of several objections to the description I have given, but none of them seems to resolve the paradox in a completely convincing way. Let me begin with some apparent difficulties which are not serious.

First of all, Newton's equations have singularities, and solutions can approach the boundary of the domain in finite time. But these triple collisions, blowups, and so forth, are problems for the determinist, not me. I am simply assuming smooth, nonsingu-lar equations on a compact domain as my definition of determinism. In short, I am assuming a smooth global flow.

The second unreal difficulty is the *chaos problem:* sensitive dependence upon initial conditions and the related fact that for numerical computations neither the inputs nor the outputs can be given exactly but only up to a certain (arbitrarily small by assumption) error. But we are mathematicians, not physicists nor philosophers of science who are frightened off by accuracy demands of googol many orders of magnitude below the diameter of an atom. By uniform continuity of the flow, for any e > 0 and any large *T,* there exists a 8 > 0 so that initial data of 8 accuracy yield computations of *f.* accuracy through time T + 1. The e accuracy needed is only enough so that, in input and output for the Babies, e approximations to 0 and 1 can be identified and distinguished. This also takes care of the conversion of continuous to discrete states performed by the Babies. We can choose the time scale so that one time unit is enough in which to change a clear 0 to a clear 1 or vice versa.

However, closely related to the chaos problem is one that came up in discussions with another colleague. This more serious *catch-up problem* may well provide the explanation.

As it can compute with 8 accuracy for any positive 8, Laplace is a pretty good computer, but presumably increasing accuracy has an increasing cost measured in time spent calculating. Once Laplace is powered up there will be a time lag while it solves the initial-value problem to T + 1, and this lag will presumably increase as 8 gets smaller. Meanwhile, even with e fixed, 8 gets smaller as T increases. It is not clear that for any T the computation out to T + 1 can be completed before time T. Some rough estimates make this difficulty appear acute. Assuming the flow is Lipschitz, then 8 can be chosen to be e divided by the Lipschitz constant. However, the Lipschitz constant for a flow on [0, T] tends to grow exponentially with T. So with e fixed, 8 is of the order exp(- *KT)* for some positive *K.* Unless the lag of computation is o(log(l/8)), for T large the lag will grow faster than T and prediction will fail. Of course, we could build a bigger, faster computer. But the computer is part of the system and the lag grows with the size of the system, that is, the number of particles in it. Furthermore, we cannot assume computations are performed with arbitrary quickness, because the calculations are performed by motions of the particles in the system whose velocities are given by the equations of the system.

There is a possible escape that should be noted. To save the paradox we don't need to predict for all large T. We only need success for some T. So there might still be a paradox even if long-term prediction fails.

Clearly, the catch-up problem is serious. Regardless of its plausibility though, I resist it. There is an appeal to efficiency, or lack thereof, which I don't trust. The whole approach has an aroma of reality that offends my mathematical nose. It was, of course, a physicist who suggested this lag problem to me.

However, if the objection could be made rigorous it would be much more interesting than my little paradox, as it would mean that certain sorts of deterministic systems are in principle not predictable except by much larger systems.

I'm afraid that my own tentative explanation is, if anything, even uglier than the catch-up problem. It has to do not with the prediction mechanism at all, but rather with the introduction of the initial data. Imagine we "photograph" the positions and velocities of all of the particles, and engrave the photo on a metal plate, whose entry into Laplace's input slot starts the clock. The problem is that the plate itself is part of the system and so a description of it must be included as part of the initial data on the plate.

Our need for only 8-accuracy may help somewhat. By making the time lag for developing, engraving, and inserting arbitrarily small, the error between the actual positions and velocities at time $t = 0$ and their pictures on the plate can be made arbitrarily small. Notice that outside the system we can allow ourselves a godlike efficiency. In particular, we have complete control of the plate and how we insert it. So we can draw what we want on it and insert it according to our description. We still have a problem of self-reference. On the plate is a little picture of the plate, containing a picture of the picture, and so forth like the infinitely receding images in facing mirrors. However, by stopping after a finite number of rounds we introduce an error whose size presumably declines with the number of images. The plate shows N nested plates and reality has $N + 1$. Unfortunately, there is a synergistic effect with the catch-up problem. Describing the images with more detail may require more particles, thus enlarging the system and shrinking 8.

What really bothers me is that the plate, as it slides toward the input slot, may bang into particles from the original system. We could wait until there are no particles in the way before starting, but this seems to be a Maxwell's demon sort of difficulty. Before this renewed onslaught from physics, I simply retreat in perplexity.

Since I originally wrote the description of this paradox I have meandered in the vast literature on determinism and free will. An anonymous referee pointed me toward *A Primer on Determinism* by John Earman, whose interests turned out to be complementary to mine. He is mostly concerned with issues that I have assumed away, like the degree to which the Newtonian universe is really deterministic given collisions, escape to the boundary, and so on. On the other hand, Karl Popper in his lecture "Of Clouds and Clocks" considered the "nightmare of determinism," and Michael Levin described a "paradox of prediction," both more closely related to the problems I am wrestling with. Their discussions take place, however, in the context of free will. So their thinking is contaminated by human minds, as mine was before the current, purified, stainless steel version of the puzzle appeared.

Meanwhile, as I have shown this puzzle about, a consensus has developed that the catch-up problem provides the key to the solution. Charles Tresser of IBM put it in a fashion that is not only palatable but elegant: "Usually, these computers are big things analyzing much smaller things. But here, the computer Laplace is analyzing a system as big as itself (bigger even). It seems that the optimal way to reveal the future in such a case is to live it." The last epigram feels like a potential conjecture. It would be nice to turn it into an actual theorem.

References

1. Earman, John, *A Primer on Determinism,* Dordrecht: D. Reidel Publ. Co. (1986).
2. Levin, Michael E., *Metaphysics and the Mind-Body Problem,* Oxford: Oxford University Press (1979).
3. Popper, Karl R., "Of Clouds and Clocks" (1965) reprinted in *Objective Knowledge,* Oxford: Oxford University Press, (1972).

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[Added in proof: A prior, closely related work is Karl Popper's "Indeterminism in Quantum Physics and in Classical Physics, Part II," *British Journal for the Philosophy of Science* (1950) 1:173-195. In explaining why a computer cannot in principle predict its own future behavior, Popper raises several arguments which overlap with mine above, but his interpretation of the results is different from my own.]

The Mathematical Intelligencer *encourages comments about the material in this issue. Letters to the editor should be sent to the editor-in-chief, Chandler Davis.*

———Prediction **and the** Spiteful Computer[1]—— A deterministic system consists of two computers, Laplace and Baby Dostoevsky. Laplace is programmed to say at tune 7\ what Baby Dostoevsky will do at some later time T_2. Baby Dostoevsky is programmed to do at time T_2 the opposite of what Laplace has said at time TV Baby Dostoevsky's method is the obvious one. Laplace's method is to calculate the state of the system at time T_2 given the initial state; this should be possible since the system is deterministic.

I suggest resolving this paradox as follows. Laplace's program includes a description of the initial state of the system. On the other hand, Laplace's program is part of the initial state of the system. Therefore, Laplace's program has to include a description of itself. There is no reason to suppose that the constraints this requirement imposes are consistent, and this resolves the paradox.

Going further, one can say that, because the supposed initial state (or program) leads to a contradiction, in fact there is no such initial state.

This is more or less the same as some of the solutions suggested in Akin's article, but it is perhaps expressed in a more mathematical and less physical way.

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This is not a physics paradox. The physical assumptions, Newtonian determinism, and uniform continuity of phase flow, as well as the requirement that predictions be secured through detailed microphysical computation, are all unnecessary scaffolding. The crux of the paradox is that the megacomputer L. is allegedly unable to make a certain prediction, which from other considerations, it obviously should be able to make. The draconian computational protocol, coupled with an assumption of determinism, is presumably intended to secure this predictability. This is computational overkill, as can be seen from the fact that the predictions L. has to make are trivially easy. L. needs to predict its own output and Baby D.'s kneejerk response.

The catch-up problem is irrelevant to the paradox's resolution. This problem arises from the profligate stipulation that predictions are to be secured through a detailed calculation based on an exact microphysical theory. We can communicate the full force of the paradox without this extra baggage, in fact, without significant physical assumptions: Call the realistic computer in this version, R. R. sticks to the essentials; it predicts only its own output and Baby D.'s inevitable negation. This is quite easy, so R. can be a small device that is not afflicted with a catch-up problem. Both are incapable of making the prediction in the required form, owing to Baby D.'s simple-minded spoiler tactics.

In short, R.'s version communicates the essential paradox, and is not tied to any particular physical theory; any possible world with enough stability for the construction of simple machines would suffice.

Not a physics paradox, this is a logical paradox, furthermore a *semantic* one because it springs from too permissive a stance on the issue of when to permit one thing to be "about" another thing. (The clearest example is Epimenides's paradox: "This sentence is false.") In Akin's paradox, L.'s output is a prediction *about* Baby D.'s response. Under the terms of the problem, specification of Baby D.'s response is tantamount to specification of L.'s prediction. Thus, L.'s prediction makes an indirect statement about itself. This dooms the prediction to be false under the specified semantic assignment. It is a lesson of modern logic that whereas rigorous use of self-application can be a wellspring, unbridled use is sure to generate paradox and self-contradiction.

L. can make the needed prediction and, say, store it in memory or relay it through a channel that Baby D. does not monitor. There is no failure of predictability, and, hence no conflict with determinism. But L. has a semantic difficulty; it cannot, without falling into error, have a certain one of its outputs *mean* (or represent or be about) its prediction.

L. is not precluded from making a valid prediction; it is precluded from expressing its prediction in a certain

[1] **See *Mathematical Intelligencer*, vol. 14, no. 2, 45-47**

manner. If L. were a human whose predictive utterances were monitored by Baby D., we might say that L. can know the truth (on the matter of Baby D.'s response), but cannot speak it.

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I think the self-reference in the input, in spite of being an obstacle, is not a valid objection, as J. von Neumann showed when he gave descriptions of self-reproductive automata. My opinion is that the solution is in the catch-up problem. For every T *»* 0, let/(T) be the time needed by Laplace to predict the output of Baby Dostoevsky at time T + 1. The paradox of the spiteful computer is just a simple proof of /(T) > T for every *T ^ 0. H* looks like a diagonalization argument against the existence of a "universal future predictor." This seems convincing to me.

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From my own "classical physicist's" point of view, *storing* numerical data incurs a cost which increases, logarithmically, with desired accuracy. (Storing D digits of information requires space, time, and expense of order D.) With chaotic dynamics, this cost increases further, logarithmically with time. A classical computer can neither contain an accurate description of its state nor predict its own future. This mechanistic picture avoids the paradox of Ethan Akin's *'Spiteful computer.*

Feedback differs from *prediction* in *influencing* the future rather than foretelling it. Thus, there is no predictive paradox in Lee Lorenz's wonderful portrayal of "Self-Awareness," from the 25 May 1992 *New Yorker.* Predicting the future is impossibly hard, while influencing it is easy. Useful to keep in mind in an election year!

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-Akin Replies-

These letters confirm my experiences discussing this puzzle. Everyone says that the problem is simple, but the proposed answers display considerable variety.

Certainly, this is a conceptual puzzle and not a physics paradox. Contra Lerma, I have reluctantly concluded that the catch-up problem is not the answer. This does not mean that for a computer to predict its own output is the trivial task that Eckhardt suggests. Such self-prediction is the heart of the paradox. The separation of the system into Laplace and Dostoevsky is just a convenient portrayal.

My residual fondness for the catch-up problem comes from its suggestion that relative size is the binding constraint against successful prediction. Such a result would free me from the *Walden Two* nightmare: My fear that something of roughly my size and complexity, for example, B. F. Skinner, could predict, and so

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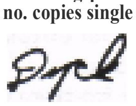

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Drawing by Lorenz; © 1992 The New Yorker Magazine, Inc.

control, my behavior. I am less bothered by predictability by something vastly greater than myself, for example, an angel.

Notice that as long as Skinner does not inform me of his predictions, his control of me does not appear to raise any more logical contradiction than does his training of any other pigeon. He merely adjusts, unknown to me, parameters whose effects on my behavior he can, by assumption, predict. The catch-up result, or Hoover's storage-size variant, would suggest that the paradox reveals a limitation upon Skinner which would deny the possibility of even such non-paradoxical control.

The trouble is that the paradox can be reconstructed with gadgets which dearly do admit a kind of self-description. Although I originally described it using a finite array of particles, the puzzle remains in force even if the computers are infinite. For infinite computers certain kinds of self-description and even self-prediction are possible.

Let {C,: i = 0, 1, . . .} be a sequence of finite computers increasing in size so that computer C, can predict by time T (fixed throughout) the results of any *T + 1* computation by any hookup of the earlier C;'s. The infinite computer is the union of the C,'s with each receiving inputs only from the programmer and the previous ones in line. Give C_0 a problem, Q the problem of predicting Q, Q the problem of predicting C,, etc. After completing its task, each component just keeps printing the same output. At time T the output is the sequence:

(working, Q says Ans, Q says CQ says Ans, . . .),

whereas at time T + 1 the output is

(Ans, C_0 says Ans, C, says C_0 says Ans, . . .).

Thus, the subsystem, {Q, C_2, . . .}, does predict the outcome of the entire system but only provided the feedback necessary to exploit the prediction does not exist.

We are left with the issue of self-reference which, I think, holds the key. However, I disagree with Eckhardt's semantic analysis. Questions about the meaning and reference of such terms as "prediction" and "about" have to do with our interpretation, from the outside, of part of the wiring diagram of the system. Such metalanguage is not required by the computers themselves. I think Steiner has it right.

My Math Department colleague Stanley Ocken agrees, although phrasing it differently. He suggests that the problem is not well-posed in that my ideal-gas particles fog over the issue of setting the whole system up. He challenges me to state the paradox in terms of finite-state machines. I do not see how to do so but that may not be significant. My imaginative facility with computers collapses long before it is hamstrung by logic.

Ocken also suggests an alternative route of escape from my Skinnerian nightmare. Complexity theory implies that for many problems, like iteration of a function, methods which exploit size superiority, like parallel processing, cannot be used to compress the number of steps which must be performed in sequence to obtain a solution. (So to build my infinite computer above, we require a sequence of machines of increasing speed rather than size. No problem. Since we are using an infinite number of components anyway, there is no reason to feel bound by the speed of light, either. But back to our universe.) "That means," I told him, "that not only can Skinner not predict me but angels can't either. Of course, God still can because he is exempt from all these rules." "That's right," was his reply. When I looked startled at his certainty, Stanley, who is Orthodox, smiled and added, "I have other sources of information."

Postscript: The article exhibited a sample of unpredictability, human or computer. The cartoon illustrating the relationship between Laplace and Dostoevsky was misattributed. It is the work of Samuel Vaughan of Berkeley, California.

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Letters to the Editor

The Mathematical Intelligencer *encourages comments about the material in this issue. Letters to the editor should be sent to the editor-in-chief, Chandler Davis.*

• Prediction Paradox

I was surprised that so many readers who wrote about the two-computer prediction paradox *[Mathematical Intelligencer,* vol. 15, no. 2, 1993, 3-5)] seemed unaware of how often this has been analyzed by logicians and philosophers of science. John Kemeny, in *A Philosopher Looks at Science* (1959), was one of the first to discuss it. Since then many variations have been proposed.

Prediction paradoxes can arise whenever a prediction is part of the event being predicted. A variation I myself invented is in Chapter 11 of my *New Mathematical Diversions from Scientific American.* You write on a sheet of paper, turning it face down so no one can read it, a description of an event that is certain to occur within an hour. You then bet a million dollars to a dime that a person cannot accurately predict whether the event will or will not occur by writing "yes" if he thinks it will, "no" if he thinks it won't. You are sure to win because the event you described is that the person will write "no."

As I pointed out, the simplest variant of this paradox is to say to someone, "Will the next word you speak be 'no'? Please answer yes or no." Such paradoxes are cleverly disguised forms of the old liar paradox which arises whenever a language is allowed to talk about the truth or falsity of its own statements.

Two notorious prediction paradoxes are much harder to resolve. One is the problem of the unexpected hanging, discussed in the first chapter of my *Unexpected Hanging.* The other is Newcomb's paradox, the topic of two chapters in my *Knotted Doughnuts.*

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Ethan Akin Responds

Gestalt psychologists have produced a number of drawings which, like the duck-rabbit, yield conflicting interpretations depending upon how they are viewed. My intent with "The Spiteful Computer" was to produce a conceptual analogue of these ambiguous figures. As a matter of logic, we interpret the setup using words like "prediction" and discover an internal contradiction. As Martin Gardner points out, it is just a version of the Liar Paradox and, as he is too polite to say, it is a rather ponderous construction compared with the examples he delightfully summarizes.

Now forget all that. Look instead at the universe of moving particles which motivated Laplace's original boast. The initial state determines all future states, and Laplace, human or machine, is equipped to compute them. The question is, "Where does the logic of the first interpretation apply its *bite* to prevent the computation, routine on the second interpretation, from succeeding?" My current answer is that the difficulty lies in describing the initial conditions, in giving to Laplace the description of the world in which he (or it) has been asked the question and been given the initial conditions.

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Mathematics and History

The recent essay "Mathematics and History," by W. S. Anglin [1], was no doubt "provocative" by intent. But there are at least two places where Anglin is so seriously unjust to earlier writers that he should not be allowed to go uncorrected.