Name: ______________________________

1. (8 points) Solve the IVP $\frac{dy}{dt} - 2y = 4 - t, y(1) = 0$.

$$[e^{-2t} y]' = (4-t)e^{-2t}$$

$$e^{-2t} y = -2e^{-2t} + \frac{1}{2}te^{-2t} + \frac{1}{4}e^{-2t} + C$$

$$y = -\frac{7}{4} + \frac{1}{2}t + Ce^{+2(t-1)}$$

$$\quad\quad\quad\quad\quad\quad \overset{\shortparallel}{5/4}$$

2. (8 points) Solve the IVP $ty' + 2y = 4t^2, y(1) = 0$.

$\mu = t^2$

$$[t^2 y]' = 4t^3$$

$$t^2 y = t^4 + C$$

$$y = t^2 + C/t^2 \quad (C = -1)$$

$$y = t^2 - 1/t^2$$

3. (8 points) Solve the IVP $y' = \frac{1-2x}{y}$, $y(1) = -2$.

$$y\,dy = (1 - 2x)\,dx$$

$$\boxed{\frac{y^2}{2} = x - x^2 + C} \qquad C = 2$$

$$\boxed{y^2 - 2x + 2x^2 = 4}$$

$$\boxed{y = -\sqrt{2x - 2x^2 + 4}}$$

4. (8 points) Find the value of b for which $(ye^{2xy} + x) + bxe^{2xy}y' = 0$ is exact and then solve it using that value of b.

$$M_y = e^{2xy} + 2xye^{2xy} = (b + 2bxy)e^{2xy} = N_x$$

$$b = 1$$

$$\psi(x,y) = \frac{1}{2}e^{2xy} + \frac{x^2}{2} + C$$

5. Given the differential equation $y'' - y' - 2y = 0$.
   (a) (6 points) Find the general solution.
   (b) (2 points) Find α so that the unique solution satisfying the differential equation and initial values $y(0) = \alpha, y'(0) = 2$ approaches zero as $t \to \infty$.

(a) $y = c_1 e^{2t} + c_2 e^{-t}$

(b) $\alpha = y(0) = c_1 + c_2$

$2 = y'(0) = 2c_1 - c_2$

$2 + \alpha = 3c_1 \rightarrow \alpha = -2$

$\boxed{c_1 = 0}$

6. (8 points) Given the differential equation $t^2 y'' - 2y = 0$ when $t > 0$.
   (a) Verify that $y_1(t) = t^2$ and $y_2(t) = \frac{1}{t}$ are solutions.
   (b) Find the Wronskian of the pair from part (a)
   (c) Verify that the linear combination $y = c_1 y_1(t) + c_2 y_2(t)$ is also a solution to the differential equation for any constants c_1 and c_2.

(a) $y_1' = 2t \quad y_2'' = 2 \rightarrow t^2 \cdot 2 - 2t^2 = 0$ ✓

$y_2' = -\frac{1}{t^2} \quad y_2'' = \frac{2}{t^3} \rightarrow t^2 \cdot \frac{2}{t^3} - \frac{2}{t} = 0$ ✓

(b) $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} t^2 & 1/t \\ 2t & -1/t^2 \end{vmatrix} = -3$

(c) $y = c_1 y_1 + c_2 y_2$

$y' = c_1 y_1' + c_2 y_2'$

$y'' = c_1 y_1'' + c_2 y_2''$

$t^2 y'' - 2y = t^2 (c_1 y_1'' + c_2 y_2'') - 2(c_1 y_1 + c_2 y_2) = 0$