Math 360 Week of May 5 Sheet

1. Verify
$$\gamma'' = u''\sigma_u + u'(u'\sigma_{uu} + v'\sigma_{uv}) + v''\sigma_v + v'(u'\sigma_{uv} + v'\sigma_{vv})$$
by using the chain rule from calculus III.

2. Use the previous problem to verify the geodesic differential equations from Theorem 9.2.1 in particular show
$$\gamma'' \cdot \sigma_u = \left(\frac{d}{dt}(u'\sigma_u + v'\sigma_v)\right) \cdot \sigma_u = \frac{d}{dt}(Eu' + Fv') - \frac{1}{2}\left(E_u(u')^2 + 2F_u u'v' + G_u(v')^2\right)$$

3. Consider hyperbolic geometry, i.e. the upper-half (v, w) plane $w > 0$ with metric
$$\frac{dv^2 + du^2}{w^2}.$$
This
   (a) Find the length of the geodesic segment $\gamma_1(t) = (0, t)$, in the hyperbolic upper-half plane when $1 \leq t \leq 4$.
   (b) Find the length of the geodesic segment $\gamma_2(t) = (3\cos t, 3\sin t)$, in the hyperbolic upper-half plane when $-\frac{\pi}{4} \leq t \leq \frac{\pi}{4}$.

4. Use complex differentials to show that each of the four basic hyperbolic isometries in Section 11.2 are indeed isometries.

5. Be able to prove Proposition 11.2.3 and Theorem 11.2.4 from section 11.2 and Propostion 11.3.2 from section 11.3.

6. (Pressley): $11.1.1 - 11.1.4, 11.2.1 - 11.2.4$

7. Read the links on our webpage: 1. ”What is the Geometry of the Universe” (Quanta magazine) and 2. ”Non-Euclidean geometry” (Wikipedia) so that you can answer the question: ”What is modern geometry?” during the one-to-one oral component of our final exam.