1. It is helpful to understand all isometries from a surface to itself. In this problem we will investigate the all the isometries of the Euclidean plane to itself.

   Remember: the Euclidean plane is the surface $(u, v, 0)$ with first fundamental form

   $$I = du^2 + dv^2,$$

   i.e. $E = 1, F = 0, G = 1$. The Euclidean plane is the surface Euclid studied. It is the surface we study in our textbook "Planimetry" by Kiselev. The foundation of our studies in that book is the following definition: two figures are the "same" if one can be superimposed onto the other. In our "Elementary Differential Geometry" text we say instead: two figures are the "same" if there is an isometry of the surface taking one figure to the other. In order to truly understand the geometry of the plane (classical Euclidean geometry) we must therefore understand the isometries of the plane.

   For each of the following describe the isometry of the plane. Then verify that each is indeed an isometry by computing $f^*\langle,\rangle = \langle,\rangle$ and also by computing E, F, and G for $f \circ \sigma$ and compare to E, F, and G for classic Euclidean Geometry.

   (a) $f(u, v) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$.

   (b) $f(u, v) = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$.

   (c) $f(u, v) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

   HINT: You may want to play around with geogebra by following the links on our webpage.

2. Verify that each of the following are conformal maps of the Euclidean plane (without the origin) to itself by by computing $f^*\langle,\rangle = \lambda\langle,\rangle$ and also by computing $\lambda E, \lambda F$, and λG for $f \circ \sigma$ and compare to E, F, and G for classic Euclidean Geometry.

   (a) $f(u, v) = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$. (HINT: You may want to play around with geogebra by following the links on our webpage.)

   (b) $f(u, v) = (u^2 - v^2, 2uv)$.

   (c) $f(u, v) = (\frac{u}{u^2+v^2}, \frac{-v}{u^2+v^2})$.

3. (Kiselev): $225, 227, 230, 239, 240, 243, 244$.

4. (Pressley): $6.3.1, 6.3.2, 6.3.4$