1. $(2, 3)$. rows are intersecting lines in the plane. linear combinations of columns span whole plane.

2. no solutions. rows are parallel lines. linear combination of columns do not span plane.

3. infinite number of solutions of form $(5 + t, -t)$ for any real number t. rows are parallel lines. linear combination of columns do not span plane.

4. substitute $(x_1, x_2) = (1, 1)$ to verify. substitute $(x_1, x_2) = (2, 0)$ to verify that it is "almost" a solution. "almost" solutions are no problem in linear algebra but in numerical linear algebra they can lead to trouble. After a computer rounds an answer it may happen that an "almost" solution is mistaken for a real solution. rows are not parallel. linear combinations of columns span the whole plane.

5. (a) Find $\|x\|_1 = 6.6, \|x\|_2 \approx 3.9573$, and $\|x\|_\infty = 3$ when $x = \begin{bmatrix} 2.1 \\ -3 \\ 1.5 \end{bmatrix}$.

   (b) Find $\|A\|_1 = 15$, and $\|A\|_\infty = 8$ when $A = \begin{bmatrix} 1 & 5 & 1 \\ -1 & 3 & -3 \\ 1 & -7 & 0 \end{bmatrix}$.

6. Find all the eigenvalues of each of the following square matrices A. Then find a basis for each eigenspace and diagonalize A by factoring $A = X\Lambda X^{-1}$, if possible.

   (a) $A = \begin{bmatrix} 7 & 8 \\ 0 & 9 \end{bmatrix}, \Lambda = \begin{bmatrix} 7 & 0 \\ 0 & 9 \end{bmatrix}, X = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 6 & 3 \\ 2 & 7 \end{bmatrix}, \Lambda = \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix}, X = \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 4 & 5 \\ -2 & -2 \end{bmatrix}$ has no real eigenvalues or real eigenvectors.

7. (a) $\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} = A = X\Lambda X^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix} = Q\Lambda Q^T$ when $Q = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, Q^T = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$ and $\Lambda = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$.

   (c) $A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.