1. Find the Lagrange polynomials $L_j(x)$ and data y_j to interpolate $f(x) = e^x$ at the x-values $x = 0, x = \frac{1}{2}$, and $x = 1$ by a quadratic polynomial $p(x)$.
   (a) $p(x) = y_0L_0(x) + y_1L_1(x) + y_2L_2(x) = 1L_0(x) + e^{0.5}L_1(x) + eL_2(x)$.
   (b) $L_0(x) = \frac{(x-\frac{1}{2})(x-1)}{(0-\frac{1}{2})(0-1)}$
   (c) $L_1(x) = \frac{(x-0)(x-1)}{(\frac{1}{2}-0)(\frac{1}{2}-1)}$
   (d) $L_2(x) = \frac{(x-0)(x-\frac{1}{2})}{(1-0)(1-\frac{1}{2})}$

2. You should be able to redo the previous exercise with two data points or four data points. Try it!
   (a) Data points: $(2, 4), (5, 1)$. Then $p(x) = -x + 6$ in all three methods.
   (b) Data points: $(-1, 3), (0, -4), (1, 5), (2, -6)$. Then $p(x) = -6x^3 + 8x^2 + 7x - 4$.
   (c) The problem above with four data points illustrates that on an exam there will never be more than four data points. It is too much computation.

3. Let $p(x)$ be the interpolating polynomial of the data points $(1, 10), (2, 10), (3, 10), (4, 10), (5, 10)$, and $(6, 15)$. Evaluate $p(7)$. ANSWER: $p(7) = 40$.

4. (a) Assume $p(x)$ interpolates $f(x) = e^{-2x}$ at the 10 evenly spaced points $x = 0, \frac{1}{9}, \frac{2}{9}, \ldots, \frac{8}{9}, 1$. An upper bound for the error $|f(\frac{1}{2}) - p(\frac{1}{2})| < 7.06 \times 10^{-11}$. How many decimal places can you guarantee to be correct if $p(0.5)$ is used to approximate e? ANSWER: 9 places.
   (b) Consider the interpolating polynomial for $f(x) = \frac{1}{x+5}$ with interpolation nodes $x = 0, 2, 4, 6, 8$, and 10. Find an upper bound for the interpolation error at $x = 1$ (ANSWER: $\frac{3\cdot5\cdot7\cdot9}{5^3}$) and then at $x = 5$ (ANSWER: $\frac{5^2\cdot3^2}{5^3}$). There are other acceptable answers.