Chapter 8 Sheet

1. Let
$$A = \begin{bmatrix} -1 & 2 \\ -1 & -4 \\ -3 & 9 \end{bmatrix}$$

(a) Find the eigenvalues and eigenvectors of A by hand.

 $\begin{bmatrix} 2\\ -2\\ 7 \end{bmatrix}$.

- (b) Use the power method by hand with initial guess $v_0 = (1, 0, 0)$ to find v_1 and λ_1 , an approximation to the dominant eigenvector and eigenvalue.
- (c) Use the inverse power method by hand with $\alpha = 0$ and with initial guess $v_0 = (1, 0, 0)$ to find v_1 , an approximation to the eigenvector corresponding to the smallest eigenvector.
- (d) Use the inverse power method by hand with $\alpha = -1$ and with initial guess $v_0 = (1, 0, 0)$ to find v_1 , an approximation to the eigenvector with corresponding eigenvalue closest to -1.

2. Let
$$A = \begin{bmatrix} -1 & 2 & 2 \\ -1 & -4 & -2 \\ -3 & 9 & 7 \end{bmatrix}$$
.

- (a) Use the power method with initial guess $v_0 = (1, 0, 0)$ to find v_{11} and λ_{11} .
- (b) Use the inverse power method with $\alpha = 0$ and with initial guess $v_0 = (1, 0, 0)$ to find v_9 and λ_9 .
- (c) Use the inverse power method with $\alpha = -1.7$ and with initial guess $v_0 = (1, 0, 0)$ to find v_8 and λ_8 .
- 3. Let $A = \begin{bmatrix} 7 & 4 \\ 3 & 6 \end{bmatrix}$.
 - (a) Find the eigenvalues and eigenvectors of A by hand.
 - (b) Use the power method by hand with initial guess $v_0 = (1, 1)$ to find v_1 and λ_1 , an approximation to the dominant eigenvector and eigenvalue.
 - (c) Use the inverse power method by hand with $\alpha = 0$ and with initial guess $v_0 = (1, 1)$ to find v_1 and λ_1 , an approximation to the smallest eigenvalue and its corresponding eigenvalue.

4. Let
$$A = \begin{bmatrix} 7 & 4 \\ 3 & 6 \end{bmatrix}$$
.

- (a) Use the power method by hand with initial guess $v_0 = (1, 1)$ to find v_{18} and λ_{18} , an approximation to the dominant eigenvector and eigenvalue.
- (b) Use the inverse power method by hand with $\alpha = 0$ and with initial guess $v_0 = (1, 1)$ to find v_{13} and λ_{13} , an approximation to the smallest eigenvalue and its corresponding eigenvector.
- 5. Find the singular values of each of the following matrices.

(a)
$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

(b)
$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

(c)
$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

6. Find the SVD of each of the following matrices.

(a)
$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

(b) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$
(c) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

7. textbook exercises: 2, 3.

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