

1. Find all solutions to the nonsingular system $\begin{cases} x_1 - 4x_2 = -10 \\ \frac{1}{2}x_1 - x_2 = -2 \end{cases}$
 - (a) Describe the Matrix Form, the Row Picture geometrically.
 - (b) the Column Picture geometrically. Do linear combinations "give" whole plane? What is the nullspace?
2. Find all solutions to the singular system $\begin{cases} x_1 + x_2 = 5 \\ 3x_1 + 3x_2 = 16 \end{cases}$
 - (a) Describe the Matrix Form, the Row Picture geometrically.
 - (b) the Column Picture geometrically. Do linear combinations "give" whole plane? What is the nullspace?
3. Find all solutions to the singular system $\begin{cases} x_1 + x_2 = 5 \\ 3x_1 + 3x_2 = 15 \end{cases}$
 - (a) Describe the Matrix Form, the Row Picture geometrically.
 - (b) the Column Picture geometrically. Do linear combinations "give" whole plane? What is the nullspace?
4. Find all solutions to the "almost" singular system $\begin{cases} 1.0001x_1 + x_2 = 2.0001 \\ 3x_1 + 3x_2 = 6 \end{cases}$
 - (a) Verify $(1, 1)$ is the only solution. Yet, $(2, 0)$ is "almost" a solution and far from $(1, 1)$. Explain. (This is no problem in linear algebra. It is a huge problem in numerical linear algebra.)
 - (b) the Column Picture geometrically. Do linear combinations "give" whole plane? What is the nullspace?
5. (a) Find $\|x\|_1$, $\|x\|_2$, and $\|x\|_\infty$ when $x = \begin{bmatrix} 2.1 \\ -3 \\ 1.5 \end{bmatrix}$.
 (b) Find $\|A\|_1$, and $\|A\|_\infty$ when $A = \begin{bmatrix} 1 & 5 & 1 \\ -1 & 3 & -3 \\ 1 & -7 & 0 \end{bmatrix}$.
6. Find all the eigenvalues of each of the following square matrices A . Then find a basis for each eigenspace and diagonalize A by factoring $A = X\Lambda X^{-1}$, if possible.
 - (a) $A = \begin{bmatrix} 7 & 8 \\ 0 & 9 \end{bmatrix}$
 - (b) $A = \begin{bmatrix} 6 & 3 \\ 2 & 7 \end{bmatrix}$
 - (c) $A = \begin{bmatrix} 4 & 5 \\ -2 & -2 \end{bmatrix}$
7. (a) Decompose $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ as $A = X\Lambda X^{-1}$.
 (b) Decompose the symmetric matrix $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ as $A = Q\Lambda Q^T$ when Q is orthogonal. Verify that Q is orthogonal.
 (c) Find the SVD of $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.
8. TEXTBOOK EXERCISES: 1, 2, 4b, 9, 10a, 11.