

1. Find the Lagrange polynomials $L_j(x)$ and data y_j to interpolate $f(x) = e^x$ at the x-values $x = 0$, $x = \frac{1}{2}$, and $x = 1$ by a quadratic polynomial $p(x)$.
2. Find the interpolating polynomial for the data points $(0, 1)$, $(2, 2)$, and $(3, 4)$ using
 - (a) Lagrange polynomials (section 10.3).
 - (b) Monomial interpolation (section 10.2).
 - (c) Divided differences (section 10.4).
 - (d) Verify that all answers above are the same polynomial.
3. You should be able to redo the previous exercise with two data points or four data points, although it does get tedious. Try it!
 - (a) Data points: $(2, 4)$, $(5, 1)$.
 - (b) Data points: $(-1, 3)$, $(0, -4)$, $(1, 5)$, $(2, -6)$.
4. Let $p(x)$ be the interpolating polynomial of the data points $(1, 10)$, $(2, 10)$, $(3, 10)$, $(4, 10)$, $(5, 10)$, and $(6, 15)$. Evaluate $p(7)$.
5.
 - (a) Assume $p(x)$ interpolates $f(x) = e^{-2x}$ at the 10 evenly spaced points $x = 0, \frac{1}{9}, \frac{2}{9}, \dots, \frac{8}{9}, 1$. Find an upper bound for the error $|f(\frac{1}{2}) - p(\frac{1}{2})|$. How many decimal places can you guarantee to be correct if $p(0.5)$ is used to approximate e ?
 - (b) Consider the interpolating polynomial for $f(x) = \frac{1}{x+5}$ with interpolation nodes $x = 0, 2, 4, 6, 8$, and 10 . Find an upper bound for the interpolation error at $x = 1$ and then at $x = 5$.
6. Write code to compute the interpolation polynomial by divided differences. This is a good exercise in using data structures (the data points) and loops. Although it is not particularly challenging code to write, you may however consider this exercise as extra credit. I will never ask you to code interpolation on an exam or homework quiz. We do not have enough time in the course. You can consider this as an extra and fun problem worth no credit.