Math 328 Exam 2 SOLUTIONS

If you run out of room for an answer, continue on the back.

Name: ______________________________

1. (10 points) Given $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.
   (a) Find $||A||_\infty$
   (b) Find the spectral radius of A.
   (c) Find all fixed points u^* of the linear iterative system $u^{(k+1)} = Au^{(k)}$.
   (d) Is the linear iterative system $u^{(k+1)} = Au^{(k)}$ stable? Use part (b) to verify your answer.

   ANSWER:
   (a) 3
   (b) 3
   (c) all points on line $y = x$.
   (d) no the fixed points are each unstable since the spectral radius is greater than 1.

2. (10 points) Factor $A = QR$ when $A = \begin{bmatrix} 2 & -2 & 18 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$.

   ANSWER: $A = QR = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 & 12 \\ 0 & 3 & -12 \\ 0 & 0 & 6 \end{bmatrix}$.

3. Find and classify as stable (or unstable) all fixed points of the linear iterative system $u^{(k+1)} = Tu^{(k)}$ when T is the matrix $T = \begin{bmatrix} .6 & .2 \\ .2 & .6 \end{bmatrix}$.

   ANSWER: There is only one fixed point $u^* = (0,0)^T$. It is stable since the spectral radius is $0.8 < 1$.

1. (10 points) Approximate solutions to the system $Ax = b$ using the Jacobi iteration method with $u^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ when $A = \begin{bmatrix} 3 & -1 & 1 \\ 3 & 6 & 2 \\ 3 & 3 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$.
   (a) Find $u^{(2)}$
   (b) Find $u^{(16)}$
   (c) Use parts a) and b) to approximate a solution.

   ANSWER:
   (a) $\begin{bmatrix} 0.14285714 \\ -0.35714286 \\ 0.42857143 \end{bmatrix}$
   (b) $\begin{bmatrix} 0.03508768 \\ -0.23684257 \\ 0.65789422 \end{bmatrix}$
   (c) something close to $u^{(16)}$ is acceptable since it seems that this choice of seed converges in Jacobi iteration.

2. (10 points) Given $A = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$. Use 8 iterations of the power method with $u^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to approximate λ_1, the larger eigenvalue and its corresponding eigenvector v_1.

   ANSWER: $\lambda_8 = 4.00036625565, v_8 = (-0.999969482421875, 1.999969482421875)$

3. (5 points) Find the approximate solution $u^{(2)}$ to the system u

$$\begin{aligned} u_1^3 - 3u_1u_2^2 - 1 &= 0 \\ 3u_1^2u_2 - u_2^3 &= 0 \end{aligned}$$

   using Newton's method with $u^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   ANSWER: $u^{(4)} = \begin{bmatrix} -0.50820704 & -0.97605255 \end{bmatrix}$. Here is my code:

```
import numpy as np
from numpy import array,dot
from numpy.linalg import inv

def f(u):
     f0 = u[0]**3 - 3*u[0]*u[1]**2 - 1
     f1 = 3*u[0]**2*u[1] - u[1]**3
     return array([f0,f1],float)

def fPrime(u):
    return array([[3*u[0]**2 - 3*u[1]**2, -6*u[0]*u[1]],
                  [6*u[0]*u[1], -3*u[1]**2]],float)

u = array([1,1],float)  # inital condition

for k in range(10):
    u = u - dot(inv(fPrime(u)), f(u))
    print(k,u)
```