

- Show all work inside this booklet.
- Calculators and all other electronic devices must be turned off and out of sight.
- Answer all questions in Part I (70 points) and 3 questions (10 points each) from Part II.
- When you finish the exam, cross out from the second column in the chart below the two problems from Part II that you have omitted.  
*If you don't do this, the first three questions that you answer (even partially) will be graded. Otherwise do not write anything in the chart below.*

Print your name: \_\_\_\_\_

Sign your name: \_\_\_\_\_

Your section: Math 20100 \_\_\_\_\_ Your instructor \_\_\_\_\_

Part I		Part II			
Problem 1		Problem 9			
Problem 2		Problem 10			
Problem 3		Problem 11			
Problem 4		Problem 12			
Problem 5		Problem 13			
Problem 6					
Problem 7					
Problem 8					
Total Part 1		Total Part 2		<b>Total</b>	

**Part I (70 points) Questions 1 to 8. Answer all questions**

**1) (12 points)** Find the derivative  $\frac{dy}{dx}$  for each of the following:

a)  $y = \tan(\sqrt{x}) + \cos(\sqrt{x^2 + 1})$

b)  $y = x^2\sqrt{x} - \frac{9}{\sqrt{x}}$

c)  $y = \left(\frac{x^2 + 1}{x^2 - 1}\right)^3$  Simplify so that your answer is a single reduced fraction.

d)  $y = \tan^4(2x)\cos^3(x)$

**Begin your work below. If you need more space, use the back of page 1.**

**2) (16 points)** Find each integral:

a)  $\int_0^{\pi/2} \cos^4 2x \sin 2x dx$

b)  $\int \frac{12x}{\sqrt{3x^2 + 120}} dx$

c)  $\int_1^3 \left(x + \frac{2}{x}\right)^2 dx$

d)  $\int_0^{\pi/4} \frac{1 + \cos^3 x}{\cos^2 x} dx$

**Begin your work below. If you need more space, use the back of page 2.**

3) a) (4 points) Find  $\frac{dy}{dx}$  at point (2,1) on the graph of the equation  $x^2y^2 = x + 2y$ .

b) (2 points) Find all points on the graph of  $y = x^3 - 3x$  where the tangent line to the graph is parallel to the line  $y = 24x + 15$  .

**Begin your work below. If you need more space, use the back of page 3.**

4) Find each of the following limits:

a) (3 points)  $\lim_{x \rightarrow 3} \frac{x^3 - 9x}{3 - x}$       b) (3 points)  $\lim_{x \rightarrow 0} \frac{x \cos(5x)}{\tan(2x)}$       c) (2 points)  $\lim_{x \rightarrow \infty} \frac{3x^2}{50 + 10x - 5x^2}$

Begin your work below. If you need more space, use the back of page 4.

**5) (6 points)** At noon, John starts walking north from point P with speed 4 miles per hour. Two hours later, Jane starts walking east from point P with speed 5 miles per hour. How fast is the distance between John and Jane increasing at 5:00 PM?

**Begin your work below. If you need more space, use the back of page 5.**

**6) (5 points)** Use calculus to find the absolute maximum and minimum values of the function  $y = x^4 - 8x^2$  for  $x$  in the interval  $[-1, 3]$ .

**Begin your work below. If you need more space, use the back of page 6.**

**7) (7 points)**

**a)** State the definition of the derivative  $f'(x)$  as a limit.

**b)** Use that definition to find  $f'(x)$  if  $f(x) = 2 - \frac{1}{4x-1}$ .

**c)** Find an equation of the tangent line to the graph of  $f(x) = 2 - \frac{1}{4x-1}$  at point  $(1, f(1))$ .

**Begin your work below. If you need more space, use the back of page 7.**



**8) (10 points)** For the function  $f(x) = \frac{x^2 - 5}{x^2 - 9}$ , you are given (do not compute!)

$$f'(x) = \frac{-8x}{(x^2 - 9)^2} \text{ and } f''(x) = \frac{24(3 + x^2)}{(x^2 - 9)^3}$$

- a)** Find the coordinates of all intercepts, and the equations of all asymptotes, of the graph of  $y = f(x)$ .
- b)** In what intervals is the function  $f$  increasing? decreasing?
- c)** In what intervals is the graph of  $f$  concave up? concave down?
- d)** Find the coordinates of all local maxima, local minima, and points of inflection of the function  $f$ .
- e)** Sketch the graph of  $y = f(x)$ . Label all the features you found in parts a) and d).

**Begin your work below. Draw your graph on the back of page 8.**

**This is the end of Part I. You should have answered all 8 questions in this part.**

**Part II. Questions 9 to 13. Answer any 3 complete questions (10 points each).**

Please look over all 5 problems in this part and then choose the 3 that you want to work on.

After you finish this part, go back to the chart on the front page of this booklet, and cross out the two problems that you wish to omit. *If you don't do this, the first three questions that you answer (even partially) will be graded.*

**9. a)** (5 points) Use differentials (linear approximation) to find an approximation to  $\frac{1}{\sqrt{1.9^2 + 5}}$ .

Write your answer as a reduced fraction.

**b)** (2 point ) State the Mean Value Theorem.

**c)** (3 points) Suppose  $y = f(x)$  is a differentiable function with  $f(2) = 3$  and  $f'(x) < 5$  for all  $x$ . What is the largest possible value for  $f(6)$  ?

**Begin your work below. If you need more space, use the back of page 9.**

**10. a)** (4 points) Suppose function  $y = f(x)$  satisfies  $f''(x) = 12x^2 + 12x$ ;  $f(0) = 4$ ;  $f(2) = 40$ . Find  $f(1)$  by first finding the general formula for  $f(x)$ .

**b)** Find each of the following limits: (3 points each)

i)  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

ii)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x+1} - 2}{x - 3}$

**Begin your work below. If you need more space, use the back of page 10.**

11. a) Let  $F(x) = \int_{2x}^{x^2} \frac{2t}{t^2 + 1} dt$ . Find  $F'(2)$ .

b) Find the average value of the function  $f(x) = \sqrt{3x+1}$  on the interval  $[1,5]$ .

**Begin your work below. If you need more space, use the back of page 11.**

**12. a) (6 points)** Let  $f(x) = \begin{cases} \frac{x^2}{4} & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ 2x & \text{if } x > 2 \end{cases}$

i) Sketch the graph of  $y = f(x)$  for  $-4 \leq x \leq 3$ .

ii) Find  $\lim_{x \rightarrow 2} f(x)$  or explain why that limit does not exist.

iii) State the definition of continuity in terms of limits, and then use that definition to explain whether or not  $f$  is continuous at  $x = 2$ .

**b) (4 points)** Use a Riemann Sum to estimate  $\int_{x=-2}^6 (2x - x^2) dx$  by using the Midpoint Rule with 4 equal subdivisions.

**Begin your work below. If you need more space, use the back of page 12.**

**13.** A cardboard box has a square base and an open top. The four sides are made of wood that costs 2 dollars per square foot, while the base is made of aluminum that costs 25 dollars per square foot. If the volume of the box is to be 50 cubic feet, what is its minimum possible cost?

**Begin your work below. If you need more space, use the back of page 13.**

**This is the end of the exam. You should have answered all question in Part I and three questions in Part II. In the chart on the front page of this booklet, cross out the two problems in Part II that you are leaving out. *If you don't do this, the first three questions that you answer (even partially) will be graded.***