The City College Department of Mathematics
Fall 2011 Math 20100 Final Exam

CALCULATORS are NOT allowed, NO scrap paper (use sheets provided)

Part I (questions 1 to 8): Answer all questions (70 points)

1) Find $\frac{dy}{dx}$ for each of the following. Simplify (a) and (c) as indicated (4 points each).

(a) $y = x^{3/2} - \frac{9}{\sqrt{x}}$ Write the answer as a single simplified fraction

(b) $y = x^3 \cos^2(4x)$

(c) $y = \frac{2x^2}{(x^2+1)^3}$ Write the answer as a single simplified fraction

2) Find each integral (4 points each).

(a) $\int \frac{1-x^2}{x^4} dx$ (b) $\int \frac{x+4}{(x^2+8x)^2} dx$

(c) $\int x^{7/2} \sec^2(2+x^{9/2}) dx$ (d) $\int_0^{\pi/6} \sin(x)\cos^2(x) dx$

3) (4 points) For the function defined implicitly by the equation $y^5 + xy = 3$ find the derivative y' at the point $(2,1)$.

4) (4 points) Find the area of the region under the curve $y = 2\sin x$ and above the x-axis lying between the lines $x = \pi/4$ and $x = \pi/2$. Include a sketch of the region.

5) (8 points) A person 6 feet tall walks away from a street light which is 24 feet above the ground.

(a) How long is the person's shadow, when the person is standing 9 feet from the street light? Justify your answer.

(b) If the shadow is lengthening at the rate of 3 feet per second, at what rate is the person walking? Justify your answer.

6) (8 points) The concentration of a drug in a patient's bloodstream t hours after it is taken is given by

$$C(t) = \frac{0.016t}{(t+2)^2} \text{ mg/cm}^3.$$

Find the maximum concentration of the drug and the time at which it occurs.

7) (8 points)

(a) State the limit definition of the derivative.

(b) Using the limit definition, compute $f'(x)$ for $f(x)=1-\frac{3}{x}$.

(c) What is the equation of the tangent line to the graph of $y=1-\frac{3}{x}$ at the point $(-1,4)$?

8) (10 points) Consider the function $f(x)=\frac{x^2}{(x-4)^2}$. Given that $f'(x)=\frac{-8x}{(x-4)^3}$ and $f''(x)=\frac{16(x+2)}{(x-4)^4}$:

(a) Find all horizontal and vertical asymptotes of the graph of $y=f(x)$.

(b) Find all critical points of $f(x)$ and the location of all local maxima and minima.

(c) Find all inflection points.

(d) Sketch the graph of $y=f(x)$, including all features determined in parts (a) to (c).

END OF PART I

Part II: Answer 3 complete questions (10 points each).

9) (a) For $f(x)=\int_0^{\sin x}(t^2+\sqrt{t})dt$, $0\le x\le\frac{\pi}{2}$, find $f'(x)$ and $f'(\pi/4)$

(b) Find $f(x)$ if $f''(x)=48x^2+6x+2$, as well as $f(1)=4$ and $f'(1)=-5$.

10) Find each of the following limits, if it exists.

(a) $\lim_{x\to 3}\frac{x^2+x-12}{x-3}$

(b) $\lim_{x\to 0}\cos\left(\frac{x^2+2\pi}{2}\right)$

(c) $\lim_{x\to\infty}\frac{x^2}{x^2-4x+3}$

(d) $\lim_{x\to 0}\frac{\sin(4x)}{\sin(6x)}$

11) A particle traveling on a straight line has the position $s(t)=8\sin(t)$ meters after *t* seconds.

(a) Find the position, the velocity and the acceleration after *t* seconds.

(b) In what direction is the particle moving at time $t=\frac{2}{3}\pi$ seconds? Is it speeding up or slowing down? Explain your answers.

(c) What is the total distance the particle travels in the time interval $\left[0,\frac{2}{3}\pi\right]$?

12) (a) Use a linear approximation to estimate $\sqrt{24.9}$ to 2 decimal places.

(b) Let $f(x)=\sqrt{x}$. Find a value of *c* satisfying the conclusion of the Mean Value Theorem for $f(x)$ on the interval $[16,25]$.

13) Consider a box with a square base of side length *s* and height *h*. If the sum of the length, width, and height of the box is 120 inches, what is the maximum possible volume? Justify your answer.

END OF EXAM