The City College Department of Mathematics
Spring 2010 Math 20100 Final Exam

Part I (70 points) Answer all questions.

1. (15pts) Find the derivative and simplify (write without negative exponents and factor if possible):
   a. $y = x^2 \sin(3x)$
   b. $f(x) = \sqrt{x} + \dfrac{3}{x^2} - 2$
   c. $y = \cos^3(x^2 + 2)$

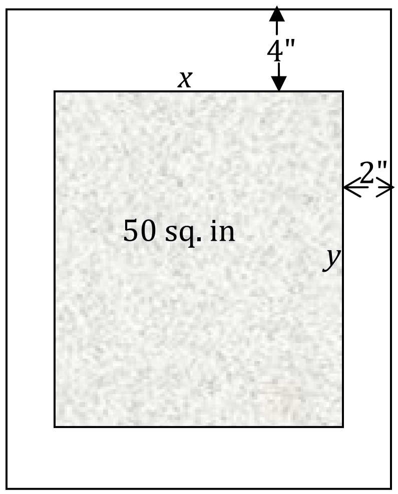
2. (15 pts) Compute each of the following integrals and simplify (write without negative exponents and compute any trig evaluations):
   a. $\int \left(\sqrt{x} + \dfrac{3}{x^2} - 2\right) dx$
   b. $\int \dfrac{6x^2 + 8}{\left(x^3 + 4x\right)^3}\, dx$
   c. $\int_{\pi/4}^{\pi/3} \sin(5x) dx$

3. (6pts) Find $f'(2)$ as a reduced fraction, where $f(x) = \dfrac{\sqrt{x^2+5}}{x^3 - x}$. (You do not have to simplify the expression for $f'(x)$).
4. (6pts) For $x^3 + y^2 = 2x^2 y + 2$
   a. Find $\dfrac{dy}{dx}$.
   b. Find the slope of the curve at $(-1,-1)$.
5. (6pts) Find the area under the graph of $y = \sin x$ over the interval $[\pi/4, \pi]$.
6. (7pts) For $f(x) = \dfrac{1}{x^2}$,
   a. Use the limit definition to find $f'(x)$.
   b. Find the equation of the tangent line to $f(x)$ at $x = -2$.
7. (7pts) When a circular plate is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50cm? Be sure to include units in your answer.
8. (8pts)Sketch the graph of $f(x) = \dfrac{x^2}{x^2 - 4}$, given that $f'(x) = \dfrac{-8x}{\left(x^2 - 4\right)^2}$ and

   $f''(x) = \dfrac{8\left(3x^2 + 4\right)}{\left(x^2 - 4\right)^3}$. Be sure to find and clearly label intercepts, horizontal and vertical asymptotes, local maxima and local minima, and inflection points, if any.

Part II (30 points) Answer 3 of the following 5 questions (10 points each). You may NOT mix and match parts a) and b) from different questions.

9. You are designing a poster with 50 sq. in. of printing, a 4 in. margin at the top and bottom, and a 2 in. margin on each side. What overall dimensions will minimize the amount of paper used? See the figure below.



10. a. Find the absolute extrema of $f(x) = x^{2/3}$ over the interval [-2,3].
    b. Use a linear approximation (or differentials) to estimate $(8.24)^{1/3}$ to two decimal places.

11. a. Find $F'(x)$ and simplify. $F(x) = \int_3^{5x^2} \sqrt{t^3+1}\, dt$
    b. Find the average value of $f(x) = 2x^2 - 1$ over [-1,2].

12. a. Use a Riemann sum with 4 equal subintervals and right endpoint evaluation points to approximate the area under the graph of $y = x^2$ over the interval [0,2]. Is this approximation an overestimate, underestimate, or neither? Explain.
    b. For the region described in a) find the exact area by taking the limit as $n \to \infty$ of a Riemann sum with *n* equal subintervals and right endpoint evaluation points. You may use the following in your computation: $\sum_{i=1}^{n} c = cn$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
    $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

13. a. Find each of the following limits, if it exists. If the limit does not exist, choose the best answer from the options $\infty, -\infty$, and DNE (does not exist).
    i. $\lim_{x \to 1} \frac{2x^2 - 2}{x^2 - 6x + 5}$

ii. $\lim_{x \to -\infty} \frac{2x^2 - 2}{x^2 - 6x + 5}$

b. For which value of k is the following function $f(x)$ continuous? Show all work.

$$f(x) = \begin{cases} 2x^2 - kx & x < 1 \\ x + 3 & x \geq 1 \end{cases}$$