The City College Department of Mathematics
Fall 2010 Math 20100 Final Exam

CALCULATORS are <u>NOT</u> allowed.

PART I: Answer ALL questions in this part. (70 points)
PART II: Answer three complete questions out of five. Each question is worth 10 points. If you answer more than three questions, cross out work you do not want graded.

Part 1 (questions 1 to 7): Answer all questions (70 points)

1)   Find $\dfrac{dy}{dx}$ (explicitly or implicitly) and simplify where possible (5 points each).

    a) $y = \cos^3(2x)$ b) $y = x^3(3x-1)^5$

    c) $y = \dfrac{x}{(2x+5)^3}$ d) $x\sin y = 1 - x^2 y$

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2)   Find each integral (5 points each).

    a) $\int (4x - 1 + 3\sqrt{x})\,dx$ b) $\int \dfrac{x}{(x^2+4)^5}\,dx$

    c) $\int_0^{1/2} x\,\sin(\pi x^2)\,dx$ d) $\int \dfrac{(x-6)^2}{x^4}\,dx$

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3)   (6 points) A particle moving in a straight line has an acceleration given by $a(t) = 2t$. The initial velocity of the particle is 2 cm/sec. How far does the particle move between $t = 1$ and $t = 2$ seconds?

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4)   (6 points) Find all the asymptotes, intercepts, maximum, minimum, and inflection points for $y = \dfrac{4x}{x^2+1}$, and sketch the graph, labeling such points. For your information $y' = \dfrac{4(1-x^2)}{(x^2+1)^2}$ and $y'' = \dfrac{8x(x^2-3)}{(x^2+1)^3}$.

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5)   (6 points) Find a) $\lim\limits_{x\to 2} \dfrac{\sqrt{x+2}-2}{x-2}$ b) $\lim\limits_{t\to 0} \dfrac{(1+\cos 2t)(1-\cos 2t)}{t^2}$

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6)   (6 points) Find the maximum and minimum values of $f(x) = x^3 + 3x^2$ on the interval $[-3,2]$.

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7) (6points) Let $g(x) = 3 + \dfrac{2}{x}$. Using the definition of derivative (no other method allowed), find
 $g'(x)$.

<div align="center">END OF PART I</div>

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Part II: Answer 3 complete questions (10 points each)

8) You have \$1200 to buy fencing, which costs \$10 per horizontal foot. The fence will surround a rectangular field and split the field down the middle, as shown in the figure to the right. What are the dimensions that will enclose the maximum possible area?

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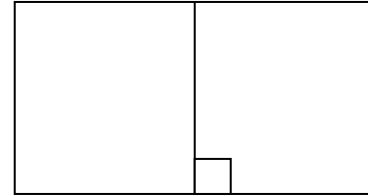
9) The picture shows car A traveling southward towards point P at 45 mi/hr. Car B is traveling east away from P at 30 mi/hr. At the instant when the distance AP is 60 mi and PB is 80 mi, what is the rate of change of distance AB?

A ● | 45 mi/hr

30 mi/hr

P | B

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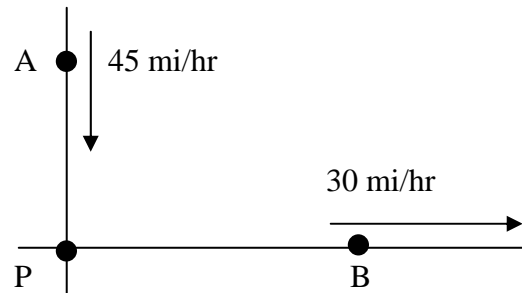
10) a) Let $F(x) = \displaystyle\int_{0}^{x} \sqrt{t^2 + 9}\; dt$. Evaluate $F''(4)$.

   b) Given $h(x) = x^{1/3}$. Find the exact value of c in the interval $(0,1)$ guaranteed by the Mean Value Theorem applied to $h(x)$.

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11) a) Use differentials to estimate $\sqrt{3.99}$.

   b) Find a point on the graph of $y = x^3 - 3x$ where the tangent line is parallel to the line $y = 9x - 8$, and write the equation of the tangent line at the point.

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12) Define $f(x) = \begin{cases} x+2 & \text{if} & x < 3 \\ 8-x & \text{if} & 3 \le x < 5 \\ 4 & \text{if} & x = 5 \\ 3 & \text{if} & x > 5 \end{cases}$

a) Sketch the graph of $f(x)$ for $0 \le x \le 7$
b) Find each of the following limits or explain why the limit doesn't exist.

i) $\lim_{x \to 3} f(x)$ ii) $\lim_{x \to 4} f(x)$ iii) $\lim_{x \to 5} f(x)$

c) Is $f(x)$ continuous at $x = 5$? Explain.