The City College of New York
Department of Mathematics
Math 20100 Final Exam Spring 2009

Instructions: No electronic device of any sort may be used during this exam. Please show all work in the space provided following each question. The time for the exam is 2 hours, 15 minutes.

Part I: Do all 10 problems (70 points)

1. Let $y = \dfrac{x^2+2}{x^2+1}$. Using rules of differentiation find y'. Express your answer as a quotient in simplified form. (5 points)

2. Let $y = x\cos^2(3x)$. Using rules of differentiation find y'. Express your answer as a sum in simplified form. (5 points)

3. Let $f(x) = \dfrac{16}{\sqrt{x}} + 3(2x)^{1/3}$. Find $f'(x)$ <u>and</u> $f'(4)$. (5 points)

4. Evaluate $\displaystyle\int \frac{2x^2 - 3\sqrt{x} + 1}{x^2}\,dx$. (5 points)

5. Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{8x+1}}$. (5 points)

6. Evaluate $\displaystyle\int \cos^4 x \sin x\,dx$. (5 points)

7a) Let $f(x) = \sqrt{x}$. State the <u>definition of the derivative as a limit</u> and use the definition to find $f'(x)$. (No credit will be given for finding the derivative using any other method.) (5 points)

b) Using the answer to 7a), what is the equation of the tangent line to the graph of $y = f(x)$ at $x = 4$? Express your answer in the form $y = ax + b$. (5 points)

8a) Let *y* be defined implicitly as a function of *x* by $y^3 + (x^2 - 1)y = 8$. Find a formula for y' in terms of *x* and *y*. (5 points)

b) Using the answer to 8a) what is the slope of the curve defined by $y^3 + (x^2 - 1)y = 8$ at the point on the curve where $x = 1$? (5 points)

9. Find the area of the region bounded by the *x* axis and the graph of $y = x^3 - 4x$ between $x = -1$ and $x = 2$. <u>Include</u> an appropriate sketch of the graph of $y = x^3 - 4x$. (10 points)

10. In the space provided, sketch a graph of $f(x)=\dfrac{4x}{x^2+1}$. Include (with justification) all intercepts, vertical and horizontal asymptotes, local maxima or minima, and inflection points. Be sure to clearly indicate the concavity. You are given that $f'(x)=\dfrac{4(1-x^2)}{(1+x^2)^2}$ and $f''(x)=\dfrac{8x(x^2-3)}{(x^2+1)^3}$. (10 points)

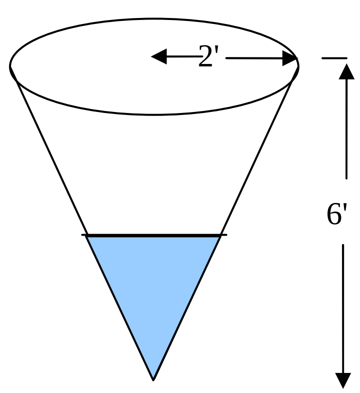
Part II: Do 3 of the following 5 problems completely. (10 points each)

11a) Use a linear approximation to an appropriate function to estimate $\sqrt{9.6}$.

b) What is the average value of the function $\cos(2x)$ on the interval $[0, \pi/4]$?

12. A closed rectangular box with a <u>square</u> base is to be constructed to hold 96 ft^3. The material for the base costs \$2 per ft^2 and the material for the top and sides costs \$1 per ft^2. Find the dimensions of the box that minimizes the cost of materials.

13. Water is draining at a rate of 2 cubic feet per minute from the bottom of a conically shaped storage tank of overall height 6 feet and radius 2 feet (see sketch below). How fast is the height of water in the tank changing when 8 cubic feet of water remain in the tank? Include appropriate units in your answer. (Note: The volume of a cone is given by $V=\frac{1}{3}\pi r^2 h$.) Your answer may be expressed in terms of π.



14a) $f(x) = \begin{cases} (x-1)^2 & \text{for } x < 1 \\ ax+b & \text{for } 1 \le x \le 4 \\ \sqrt{2x+1} & \text{for } 4 < x \end{cases}$. Find a and b so that $f(x)$ is continuous for all x.

b) Find $\lim_{x \to 0} x \csc(2x)$.

15. A small balloon starts from the ground at $t = 0$ and rises straight up with a velocity of $v(t) = \dfrac{1000}{(10+t)^2}$ ft/sec for $t \ge 0$. For simplicity, assume the balloon can be considered as a point.

a) Find the height $h(t)$ of the balloon above the ground for $t \ge 0$.
b) What happens to the height of the balloon as t increases without bound?