MATH 201 **Final Examination** **Fall 2009**

Instructions: No calculators or electronic devices may be used during the exam. You must show work justifying all answers. Answer each question in the space provided. If you need extra space please use the back of each sheet.

PART 1 (70 points) Answer all questions

1) (16 points) Find $\frac{dy}{dx}$. Simplify your answers in a) and b).

a) $y = 4x^3(2x+1)^2$ **b)** $y = \frac{2x}{\sqrt{x+8}}$

c) $y\sin x + x^2y^2 = 2x$ **d)** $y = \tan^2(x^3+2)$

2) (16 points) Compute each of the following integrals

a) $\int \frac{4x^2-4x+1}{x^{3/2}}dx$ b) $\int \sin x \cos x dx$

c) $\int \frac{3x^3}{\sqrt{x^4+1}}dx$ d) $\int_0^{\sqrt{\pi/4}} 4x\sec^2(x^2)\,dx$

3) (10 points) a) State the definition of $f'(x)$ as a limit.

b) Using the definition of the derivative, compute $f'(x)$ if $f(x) = x^2 + 2x$ (No credit will be given for any other method)

c) For the function in 3b) using the answer to b) find an equation of the tangent line to $y = f(x)$ at the point $(1,3)$.

4) (8 points) Evaluate the area of the region bounded by the graphs of the equations below. Include a sketch of the region.

$y = x^3 + x$, $x = 2$, $y = 0$

5) (10 points) For the function $f\left(x\right)=\frac{x^2-9}{(x-1)^2}$, we have $f'\left(x\right)=\frac{2(9-x)}{\left(x-1\right)^3}$ and $f''\left(x\right)=\frac{4\left(x-13\right)}{\left(x-1\right)^4}$. (You do not have to verify this.)
Using the given information, sketch the graph of $f(x)$. Include in your graph the following elements, clearly labeled and justified by appropriate calculations:
intercepts, horizontal and vertical asymptotes, local maxima and minima, and inflection points, if any:

6) (10 points) You are asked to design a rectangular container with a square base and no top. Your budget allows you to use a total of 300 ft^2. of material for the sides and bottom. What are the dimensions of the largest capacity container you can design with this constraint?

PART 2 (30 Points) Solve 3 complete problems out of 5. If you solve more than three problems please indicate which problems should be omitted from grading.

7) (10 points) a) Using differentials or linear approximations, estimate the value of $\sqrt[3]{26}$. (You may leave your answer in fractional form.).

b) Using a Riemann sum with three equal subintervals and left endpoints, estimate the value of $\int_1^4 \frac{dx}{x}$. Leave your answer as a fraction.

8) (10 points) A 25-foot long ladder is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

a) How fast is the top of the ladder moving down the wall when its base is 7 feet from the wall?
b) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.

9) (10 points) Find the absolute extrema of the function $f\left(x\right)=2x^2-\sqrt{x}$ on the interval $\left[0,1\right]$.

10) (10 points) a) Compute each of the following limits or show that the limit does not exist

i) $\lim_{x\to 0}\frac{\tan 2x}{\tan 3x}$ ii) $\lim_{x\to 1}\frac{x^2-1}{x^2+2x-3}$

b) Let $F(x)=\int_0^{3x}\sqrt{t^2+2t}\,dt$. What is the value of $F'(1)$?

11) (10 points)

a) A particle moves along a straight line path in such a way that its acceleration at any time is given by $a(t)=\sin(t)+\frac{1}{2}$. If the initial velocity is $v(0)=0$ and the initial displacement is $s(0)=0$, find formulas for the velocity and displacement at any time *t*.
b) Find a value c in the interval (1, 3) where the function $f\left(x\right)=x^{-2}$ takes on its average value on that interval.